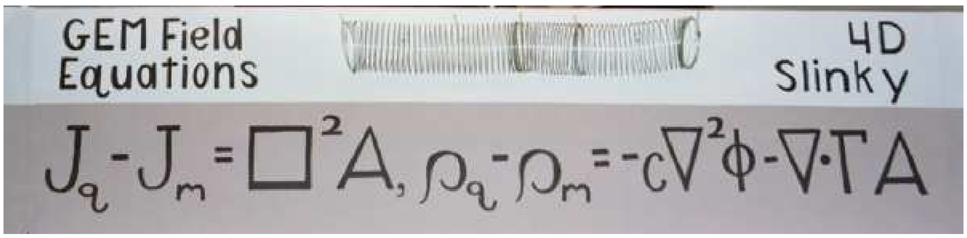
## Generalizing EM

"General Gauss' Law", electric minus inertial current equals changes in 4-potential and the connection.

$$4\pi(\rho_q - \rho_m) = \Box^2\phi = \partial^\mu A_{0,\mu} - \partial^\mu \Gamma^{\alpha}{}_{0\mu} A_\alpha$$

- $\rho_q - \rho_m$ Less total current due to inertia.
- $\rho_m = \sqrt{G}\, m/\text{vol.}$ Mass in units of electric charge ($F = -\frac{(\sqrt{G}\,m)(\sqrt{G}\,m)}{R^2}$).
- $\partial^\mu A_{0,\mu}$ Divergence of a potential gradient.
- $\partial^\mu \Gamma^{\alpha}{}_{0\mu} A_\alpha$ Divergence of the connection.

Standard EM plus a sophisticated handle for spacetime curvature.




## Geometric Structure

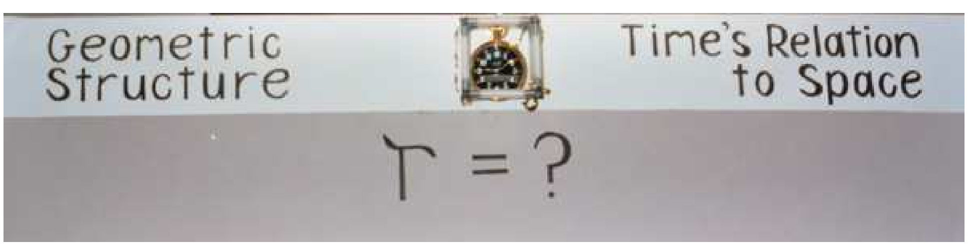
The way to determine distance, volumes and transformations in spacetime.

1. Presumed
    - Newton: time and space are absolute, no $t \leftrightarrow R$ rotations (boosts).
    - Special relativity: Only inertial observers.
2. Background chosen

    $\rho_q = g^{\mu\nu}\partial_\mu(A_{0,\nu} - A_{\nu,0})$ Maxwell: Could be any metric, you choose.
3. Background-Free

    $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi T_{\mu\nu}$ General relativity: Solve field equations for the metric.

Any theory for gravity must be background-free




## Geometry and Gen. Gauss' Law

Address a critique by Prof. John Baez in the newsgroup Sci.Physics.Research:

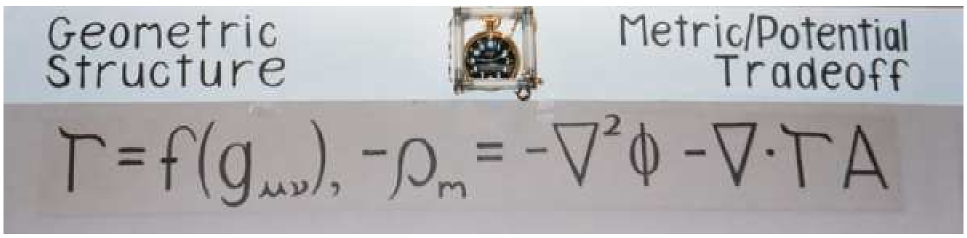
"Instead what really matters is that Sweetser's model is a field theory on Minkowski spacetime, not a background-free theory like general relativity"

Torsion-free, metric compatible connection means there is a unique metric for the connection (Christoffel symbol).

Focus on static point sources, the simplest, non-trivial case.

Disprove the critique with four calculations of charge densities:

1. Neutron in flat Euclidean spacetime.
2. Proton in flat Euclidean spacetime.
3. Neutron in curved spacetime.
4. Proton in curved spacetime.




## Neutron in Flat Euclidean Spacetime

Newton's law of gravity with units of electric charge.

- $A = (\frac{\sqrt{G}\,m}{R}, 0, 0, 0)$ 1/R potential using mass in units of electric charge ($F = -\frac{(\sqrt{G}\,m)(\sqrt{G}\,m)}{R^2}$).
- $g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$ The Minkowski metric.
- $\Gamma^{\alpha}{}_{\mu\nu} = 0$ Connection is zero in Euclidean spacetime.

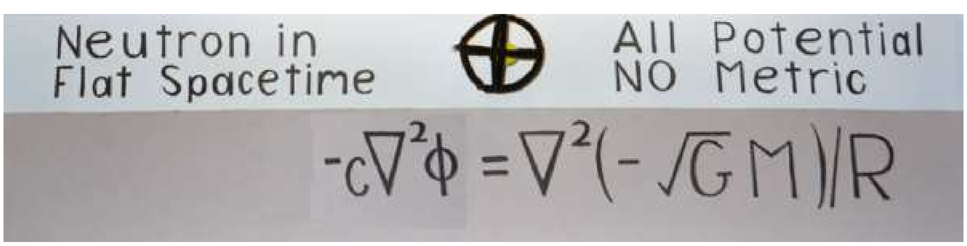
Calculate the average charge density of a neutron in a 1 cm sphere.

$$4\pi(\rho_q - \rho_m) = \frac{\text{charge}}{\text{vol.}} = \frac{-4\pi\sqrt{G}\,m}{4\pi R^3/3}$$

$$= -3\sqrt{6.67\,\text{x}\,10^{-11}\text{m}^3/\text{kg s}^2}\; 1.67\,\text{x}\,10^{-27}\text{kg}/(0.01\,\text{m})^3$$

$$= -4.09\,\text{x}\,10^{-26}\,\text{C/m}^3$$

$$\partial^i A_{0,i} - \partial^i\,\Gamma^{\alpha}{}_{0i}\,A_\alpha = \partial^i A_{0,i} = -\nabla^2\phi = \nabla^2(-\frac{\sqrt{G}\,m}{R}) = \cdots = -3\frac{\sqrt{G}\,m}{R^3}$$



## Proton in Flat Euclidean Spacetime

Newton's and Gauss' law: shared potential, different charges.

- $A = (\frac{-q + \sqrt{G}\, m}{R}, 0, 0, 0)$  1/R potential with 2 charges.
- $g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$  The Minkowski metric.
- $\Gamma^{\alpha}{}_{\mu\nu} = 0$  Connection is zero in Euclidean spacetime.

Calculate the average charge density of a proton in a 1 cm sphere.

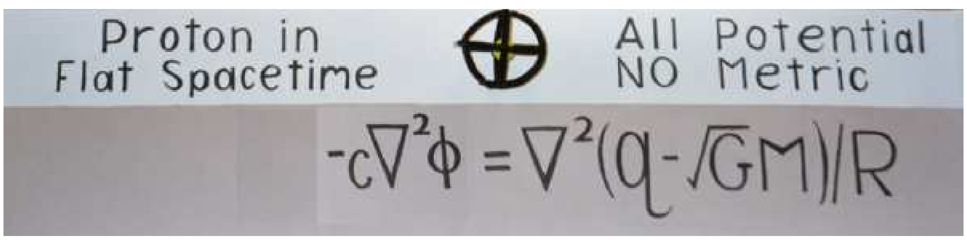
$$4\pi(\rho_q - \rho_m) = \frac{\text{charge}}{\text{vol.}} = \frac{4\pi(q - \sqrt{G}\, m)}{4\pi R^3/3}$$

$$= 3(1.60 \times 10^{-19}C - \sqrt{6.67 \times 10^{-11}\mathrm{m}^3/\mathrm{kg\,s}^2}\; 1.67 \times 10^{-27}\mathrm{kg})/(0.01\,\mathrm{m})^3$$

$$= (4.80 \times 10^{-13} - 4.09 \times 10^{-26})\ \mathrm{C/m}^3$$

$$\partial^i A_{0,i} - \partial^i \Gamma^{\alpha}{}_{0i} A_\alpha = \partial^i A_{0,i} = -\nabla^2 \phi = \nabla^2 (\frac{q - \sqrt{G}\, m}{R}) = \cdots = 3\,\frac{q - \sqrt{G}\, m}{R^3}$$

Vast difference in the size of charges.




## Neutron in Curved Spacetime

Charge density due to connection, not the potential.

- $A = (\frac{c^2}{\sqrt{G}}, 0, 0, 0)$  A constant potential with the correct units ($\frac{q}{r} \rightarrow \frac{\sqrt{kg\, m}}{s}$).
- $g_{\mu\nu} = \begin{pmatrix} e^{-2GM/c^2R} & 0 & 0 & 0 \\ 0 & -e^{2GM/c^2R} & 0 & 0 \\ 0 & 0 & -e^{2GM/c^2R} & 0 \\ 0 & 0 & 0 & -e^{2GM/c^2R} \end{pmatrix}$

  This metric is consistent with tests of weak gravity fields.

$g^{00} = e^{2GM/c^2R}$  because $g_{00}\, g^{00} = 1$.

$\nabla g_{00} = -2 e^{-2GM/c^2R} \nabla \frac{GM}{c^2 R}$  $g^{00} \nabla g_{00} = -2\, \nabla \frac{GM}{c^2 R}$

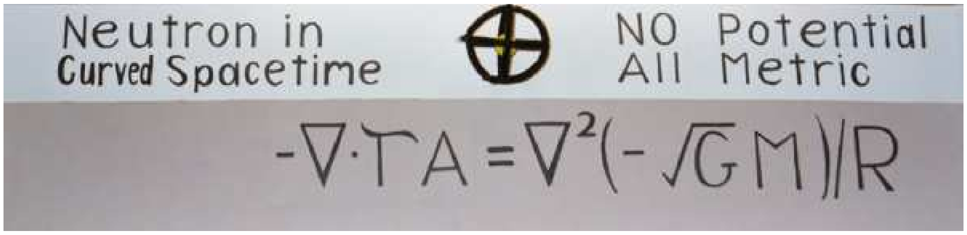
Analyze the divergence of the Christoffel symbol.

$$-\partial^i \Gamma^{\alpha}{}_{0i} A_\alpha = -\frac{1}{2} \partial^i g^{\alpha\beta}(\partial_i g_{0\beta} + \partial_0 g_{i\beta} - \partial_\beta g_{0i}) A_\alpha$$

$$\alpha = \beta = 0 \quad \text{static \& diagonal} \rightarrow 0$$

$$= \frac{1}{2}\,\nabla g^{00}\,\nabla g_{00}\,\phi$$

$$= \nabla^2\,(-\frac{\sqrt{G}\,m}{R}) = \cdots = -3\,\frac{\sqrt{G}\,m}{R^3} \qquad \text{QED}$$




### Proton in Curved Spacetime

Only the charge in the metric changes.

- $A = (\frac{c^2}{\sqrt{G}}, 0, 0, 0)$ A constant potential.
- $g_{\mu\nu} = \begin{pmatrix} e^{2(\sqrt{G}q-GM)/c^2R} & 0 & 0 & 0 \\ 0 & -e^{-2(\sqrt{G}q-GM)/c^2R} & 0 & 0 \\ 0 & 0 & -e^{-2(\sqrt{G}q-GM)/c^2R} & 0 \\ 0 & 0 & 0 & -e^{-2(\sqrt{G}q-GM)/c^2R} \end{pmatrix}$

$g^{00} = e^{-2\,(\sqrt{G}\,q-GM)/c^2R}$ $\qquad g^{00}\nabla g_{00} = 2\,\nabla\,\frac{\sqrt{G}\,q-GM}{c^2\,R}$

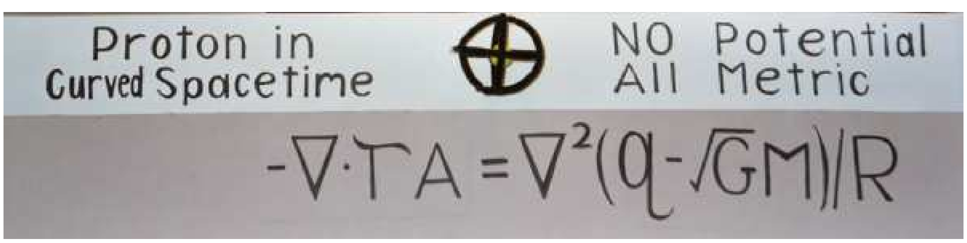
Analyze the divergence of the Christoffel symbol.

$$-\,\partial^i\,\Gamma^{\,\alpha}{}_{0i}\,A_\alpha = -\frac{1}{2}\,\partial^i g^{\alpha\beta}(\partial_i\,g_{0\beta} + \partial_0 g_{i\beta} - \partial_\beta\,g_{0i})\,A_\alpha$$

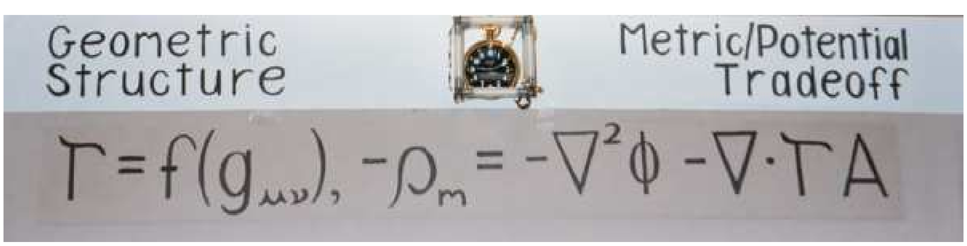
$$\alpha = \beta = 0 \quad \text{static \& diagonal} \rightarrow 0$$

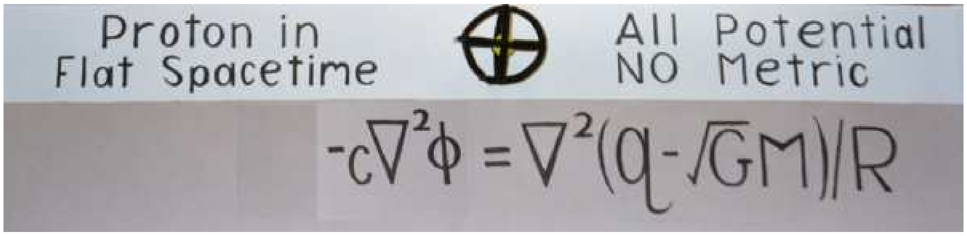
$$= \frac{1}{2}\,\nabla g^{00}\,\nabla g_{00}\,\phi$$

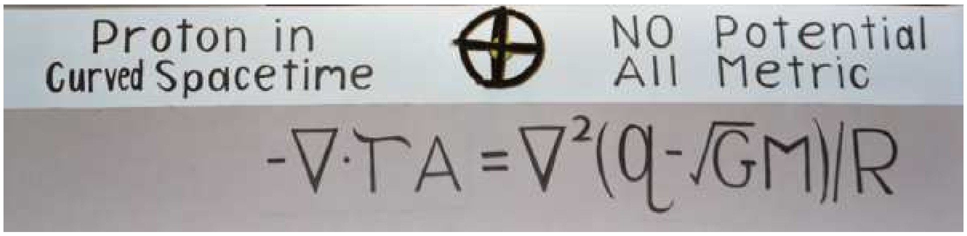
$$= \nabla^2\,(\frac{q-\sqrt{G}\,m}{R}) = \cdots = 3\,\frac{q-\sqrt{G}\,m}{R^3} \qquad \text{QED}$$




### Summary: Metric/Potential Diffeomorphism




Proton in
Flat Spacetime
All Potential
NO Metric
-c∇²ϕ = ∇²(q-√GM)/R


Proton in
Curved Spacetime
NO Potential
All Metric
-∇·TA = ∇²(q-√GM)/R