Why a Rank 1 Unified Field Theory is Compelling and Its Background Mathematical Structure

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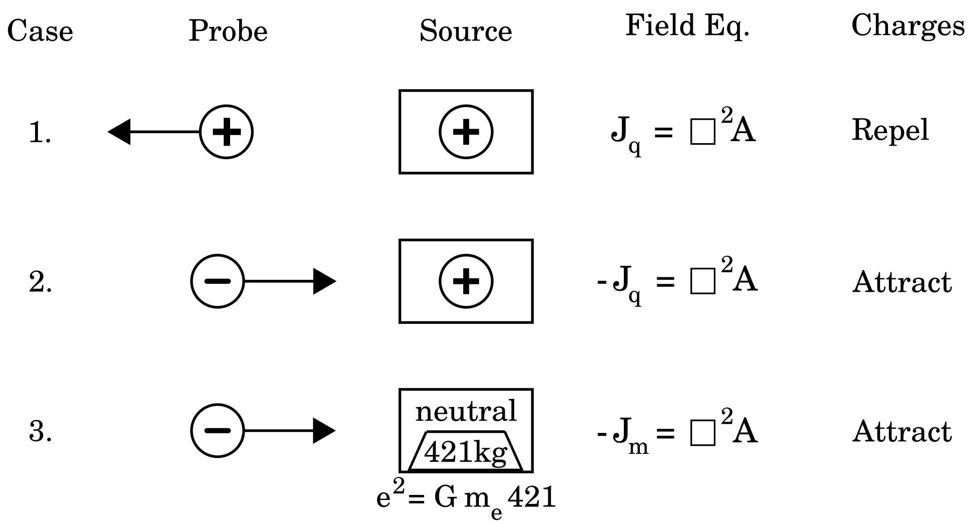
**Abstract**

It is necessary to use a rank 2 field theory to explain gravity. The constraints on a theory which unified gravity and electromagnetism are less clear. A thought experiment with point charges suggests that if Nature is logically consistent in her choice for quantizeable attractive forces, gravity and EM must be rank 1 field equations. The current split between the four known forces of Nature - gravity versus EM, weak, and the strong force - can be viewed as the way physicists have operationally dealt with the covariant 4-derivative of a 4-potential.

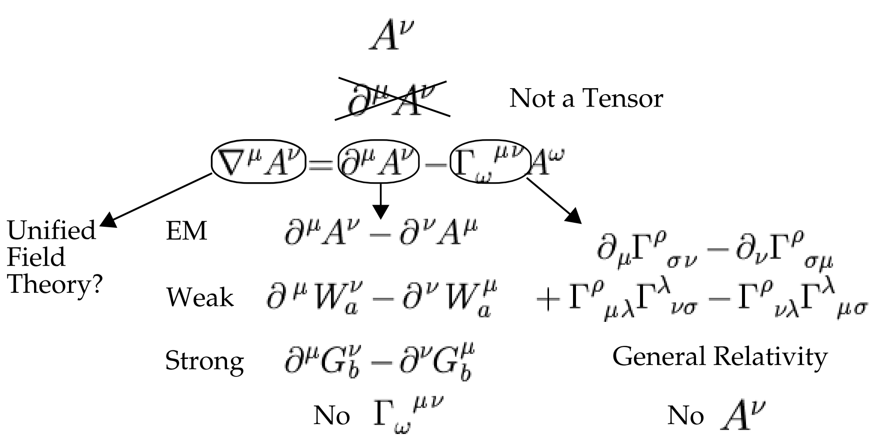
The mathematical structure for my rank 1 unified field theory is almost identical to that of the Maxwell equations. One needs a 4-dimensional differential manifold with a connection. The Maxwell equations require that the metric be provided as part of the background mathematical structure since the antisymmetric electromagnetic field strength tensor cannot constrain how the symmetric metric tensor changes. The unified field strength tensor of this proposal is asymmetric. On a Riemannian manifold, it can constrain the changes of a metric tensor up to a diffeomorphism with the change of the potential. The diffeomorphism will be shown by direct calculation.

## Attractive 4-Vector Field Equations

The same manifestly covariant law should apply in different situations:




## The 4 Forces of Nature Split

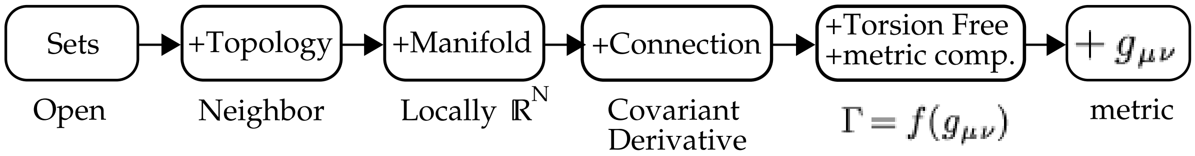



## Background Metrics

- Fixed: Newton, special relativity, current string theory.
- Chosen: Maxwell
- Determined by field equations: general relativity, GEM?

## Maxwell's Background Structure

Structure required to do calculus.

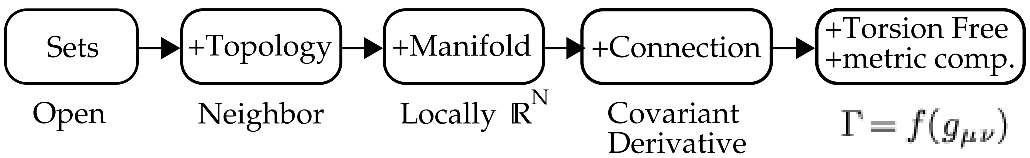



Metric *must* be supplied.

The anti-symmetric field strength tensor $F^{\mu\nu}$ can provide no information on how the symmetric metric tensor changes.

## GEM's [Gravity and EM] Background Structure

Everything but the metric.




The same background structure as general relativity.

Metric *determined* by circumstances, up to a diffeomorphism with changes in the potential.

The asymmetric field strength tensor $\nabla_\mu A^\nu$ can provide information on how the symmetric metric tensor changes.

## General Gauss' Law

For gravity and EM, using both the potential and connection.

$$4\pi(\rho_q - \rho_m) = \frac{\partial^2\phi}{\partial t} - \nabla^2\phi + \frac{\partial}{\partial t}\Gamma^0{}_{\omega 0} A^\omega - \nabla^i \Gamma^0{}_{\omega i} A^\omega$$

Two solutions for a static point source (proton at 1 cm):

Solution 1: Flat Euclidean spacetime with a dynamic $\frac{1}{R}$ potential:

- $\Gamma = 0, \nabla\Gamma = 0$
- $A^\mu = (\frac{q - \sqrt{G} m_p}{R}, 0, 0, 0)$
- $\rho = -\nabla^2 \frac{q - \sqrt{G}\, m_p}{R} = (4.80x10^{-13} - 4.09x10^{-26}) C/m^3$

Solution 2: Constant potential with an exponential $\frac{1}{R}$ metric:

- $A^\mu = (a, b, c, d), \partial_\mu A^\mu = 0$
- $$g_{\mu\nu} = \begin{pmatrix} e^{2(\frac{\sqrt{G}\, q - G m_p}{c^2 R})} & 0 & 0 & 0 \\ 0 & e^{-2(\frac{\sqrt{G}\, q - G m_p}{c^2 R})} & 0 & 0 \\ 0 & 0 & e^{-2(\frac{\sqrt{G}\, q - G m_p}{c^2 R})} & 0 \\ 0 & 0 & 0 & e^{-2(\frac{\sqrt{G}\, q - G m_p}{c^2 R})} \end{pmatrix}$$
- Because the metric is static and diagonal, only the $g_{00}$ term contributes to the divergence of the connection:

  $$-\frac{1}{2}\nabla g^{00}\nabla g_{00} = -\nabla^2 \frac{q - \sqrt{G}\, m_p}{R} = (4.80x10^{-13} - 4.09x10^{-26}) C/m^3$$