Unifying Gravity and EM by Analogies to EM

Presented by Doug Sweetser, '84, sweetser@alum.mit.edu, quaternions.com

Sponsor: John A. Fries

Short Description:

Investigate an old hypothesis, that gravity is similar to EM. Clone a Lagrange density for gravity from EM. A problem has been the distance dependence. Impress friends by deriving Newton's law of gravity using perturbations of a normalized potential. The mundane chain rule may eliminate the need for dark matter and energy. The subtle underlying idea will be discussed while Grand Marnier chocolate truffles are served.

Skeptics welcome.

Day 2: Table of Contents

Outline for Day 2

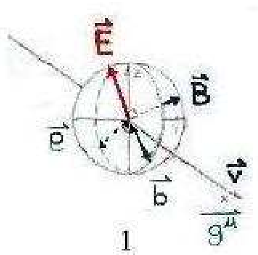
Must Do Physics.

1. Fields.

2. Quantization.

3. The Standard Model.

Must Do Physics Done.



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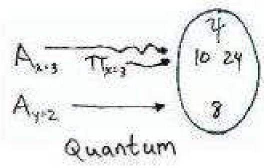

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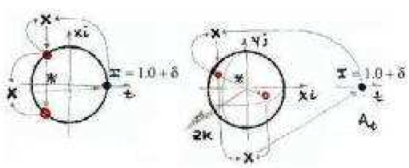

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Must Do Physics for Day 2

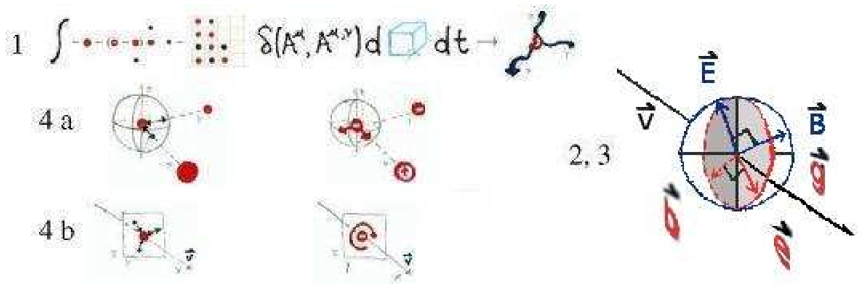
1. $F_g = -Gm\psi\hat{R}$ Like charges attract.
2. $+m$ One charge.
3. $\rho = \nabla^2\phi$ Newton's gravitational field equation.
4. $m\frac{d^2\vec{R}}{dt^2} = -\frac{GMm}{R^2}\hat{R}$ Newton's law of gravity under classical conditions.
5. $d\tau^2 = (1 - 2\frac{GM}{c^2R} + 2(\frac{GM}{c^2R})^2)\,dt^2 - (1 + 2\frac{GM}{c^2R})\,\frac{dR^2}{c^2}$

   Consistent with the Schwarzschild metric.
6. $F_{\text{EM}} = q\vec{E}$ Like charges repel.
7. $\pm q$ Two distinct charges.
8. $\rho = \vec{\nabla}\cdot\vec{E}$ $\vec{J} = -\frac{\partial\vec{E}}{c\,\partial t} + \vec{\nabla}\times\vec{B}$ Maxwell source equations.
9. $0 = \vec{\nabla}\cdot\vec{B}$ $\vec{0} = \frac{\partial\vec{B}}{c\partial t} + \vec{\nabla}\times\vec{E}$ Maxwell homogeneous equations.
10. $F^{\mu} = q\,\frac{U_{\nu}}{c}(\nabla^{\mu}A^{\nu} - \nabla^{\nu}A^{\mu})$ Lorentz force.
11. Unified field emission modes can be quantized.
12. Works with the standard model.
13. Indicates origin of mass.
14. LIGO (gravity wave polarization).
15. Rotation profiles of spiral galaxies.
16. Big Bang constant velocity distribution.

Will try to do 3, 8, 9, 11-14.

Fields

1. The Players.
2. Euler-Lagrange equation:
    a) Principle of least action.
    b) Derivation.
    c) Apply to GEM Lagrange density.
3. Classical fields.
4. Classical fields in detail.
5. Classical field equations:
    a) Gauss' law and Newton's [relativistic] gravitational field.
    b) Ampere's law and mass current.
    c) Vector identities.



The Players

A table of the players in fields and field equations. Three new fields for gravity will be introduced subsequently.

Rank	*Symbol*	*Name*
0	$\mathfrak{L}$	Lagrange density
1	A^ν	Potential
1	$c\frac{\partial\mathfrak{L}}{\partial A^\nu} = c\nabla^\mu(\frac{\partial\mathfrak{L}}{\partial\nabla^\mu A^\nu})$	Field equations
1	$\frac{\partial\vec{E}}{c\partial t}, \vec{\nabla}\times\vec{E}, \frac{\partial\vec{e}}{c\partial t}, \frac{\partial\vec{B}}{c\partial t}, \vec{\nabla}\times\vec{B}, \vec{\nabla}\boxtimes\vec{b}, \nabla g^\mu$	Field equations as classical fields
2	$\vec{E}, \vec{e}, \vec{B}, \vec{b}, g^\mu$	Classical fields which constitute $\nabla^\mu A^\nu$

Principle of Least Action

The spacetime integral of a Lagrange density:

1. The action.

   $S = \int \mathfrak{L} \sqrt{-g}\, \partial V \partial t$

2. $\delta S = \int \left(\frac{\partial \mathfrak{L}}{\partial A^\nu} \delta A^\nu + \frac{\partial \mathfrak{L}}{\partial \nabla^\mu A^\nu} \delta \nabla^\mu A^\nu\right) \sqrt{-g}\, \partial V \partial t = 0$ Minimize the action.

Search for minimal function, not value, using calculus of variations.

Derive the Euler-Lagrange Equation

Local covariant coordinates will be used in the following work.

1. Start with a Lagrange density that is a function of A^ν and $\nabla^\mu A^\nu$ (not position or velocity):

   $\mathfrak{L} = f(A^\nu, \nabla^\mu A^\nu)$

2. Form the action:

   $S = \int \mathcal{L}(A^\nu, \nabla^\mu A^\nu) \sqrt{-g}\, \partial V \partial t$

3. Take the variation of the action:

   $\delta S = \int \left(\frac{\partial \mathfrak{L}}{\partial A^\nu} \delta A^\nu + \frac{\partial \mathfrak{L}}{\partial \nabla^\mu A^\nu} \delta \nabla^\mu A^\nu\right) \sqrt{-g}\, \partial V \partial t$

4. Rewrite the 2nd term using the chain rule, subtracting the excess term:

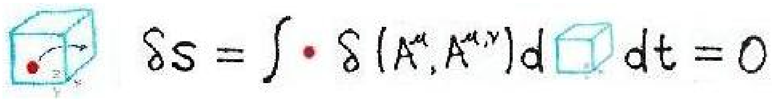
   $\delta S = \int \left(\frac{\partial \mathfrak{L}}{\partial A^\nu} \delta A^\nu + \nabla^\mu\left(\frac{\partial \mathfrak{L}}{\partial \nabla^\mu A^\nu} \delta A^\nu\right) - \nabla^\mu\left(\frac{\partial \mathfrak{L}}{\partial \nabla^\mu A^\nu}\right) \delta A^\nu\right) \sqrt{-g}\, \partial V \partial t$

5. Integral of 2nd term is zero (a theorem of Gauss):

   $\delta S = \int \left(\frac{\partial \mathfrak{L}}{\partial A^\nu} - \nabla^\mu\left(\frac{\partial \mathfrak{L}}{\partial \nabla^\mu A^\nu}\right)\right) \delta A^\nu \sqrt{-g}\, \partial V \partial t$

6. Set integral to zero, which is true for all possible variations if integrand is zero:

   $\frac{\partial \mathfrak{L}}{\partial A^\nu} = \nabla^\mu\left(\frac{\partial \mathfrak{L}}{\partial \nabla^\mu A^\nu}\right)$



Apply Euler-Lagrange to GEM Lagrange Density

1. Start with Euler-Lagrange, $\frac{\partial \mathfrak{L}}{\partial A^{\nu}} = \nabla^{\mu}(\frac{\partial \mathfrak{L}}{\partial \nabla^{\mu} A^{\nu}})$, written without indices:

$$c\,\frac{\partial \mathfrak{L}}{\partial \phi} = c(\frac{\partial}{\partial t}(\frac{\partial \mathfrak{L}}{\partial(\frac{\partial \phi}{\partial t})}) - \frac{\partial}{\partial x}(\frac{\partial \mathfrak{L}}{\partial(-\frac{\partial \phi}{\partial x})}) - \frac{\partial}{\partial y}(\frac{\partial \mathfrak{L}}{\partial(-\frac{\partial \phi}{\partial y})}) - \frac{\partial}{\partial z}(\frac{\partial \mathfrak{L}}{\partial(-\frac{\partial \phi}{\partial z})}))$$

$$c\,\frac{\partial \mathfrak{L}}{\partial A_x} = c(\frac{\partial}{\partial t}(\frac{\partial \mathfrak{L}}{\partial(\frac{\partial A_x}{\partial t})}) - \frac{\partial}{\partial x}(\frac{\partial \mathfrak{L}}{\partial(-\frac{\partial A_x}{\partial x})}) - \frac{\partial}{\partial y}(\frac{\partial \mathfrak{L}}{\partial(-\frac{\partial A_x}{\partial y})}) - \frac{\partial}{\partial z}(\frac{\partial \mathfrak{L}}{\partial(-\frac{\partial A_x}{\partial z})}))$$

$$c\,\frac{\partial \mathfrak{L}}{\partial A_y} = c(\frac{\partial}{\partial t}(\frac{\partial \mathfrak{L}}{\partial(\frac{\partial A_y}{\partial t})}) - \frac{\partial}{\partial x}(\frac{\partial \mathfrak{L}}{\partial(-\frac{\partial A_y}{\partial x})}) - \frac{\partial}{\partial y}(\frac{\partial \mathfrak{L}}{\partial(-\frac{\partial \mathrm{Ay}}{\partial y})}) - \frac{\partial}{\partial z}(\frac{\partial \mathfrak{L}}{\partial(-\frac{\partial A_y}{\partial z})}))$$

$$c\,\frac{\partial \mathfrak{L}}{\partial A_z} = c(\frac{\partial}{\partial t}(\frac{\partial \mathfrak{L}}{\partial(\frac{\partial A_z}{\partial t})}) - \frac{\partial}{\partial x}(\frac{\partial \mathfrak{L}}{\partial(-\frac{\partial A_z}{\partial x})}) - \frac{\partial}{\partial y}(\frac{\partial \mathfrak{L}}{\partial(-\frac{\partial \mathrm{Az}}{\partial y})}) - \frac{\partial}{\partial z}(\frac{\partial \mathfrak{L}}{\partial(-\frac{\partial A_z}{\partial z})}))$$

2. Write out GEM Lagrange density without indices:

$$\mathfrak{L} = -\rho_m(\sqrt{1 - (\frac{\partial x}{c\partial t})^2 - (\frac{\partial y}{c\partial t})^2 - (\frac{\partial z}{c\partial t})^2}$$

$$-(\rho_q - \rho_m)(c\phi - \frac{\partial x}{\partial t}A_x - \frac{\partial x}{\partial t}A_y - \frac{\partial z}{\partial t}A_z)$$

$$-\frac{1}{2}((\frac{\partial \phi}{c\partial t})^2 - (\frac{\partial \phi}{\partial x})^2 - (\frac{\partial \phi}{\partial y})^2 - (\frac{\partial \phi}{\partial z})^2 - (\frac{\partial A_x}{c\partial t})^2 + (\frac{\partial A_x}{\partial x})^2 + (\frac{\partial A_x}{\partial y})^2 + (\frac{\partial A_x}{\partial z})^2$$

$$-(\frac{\partial A_y}{c\partial t})^2 + (\frac{\partial A_y}{\partial x})^2 + (\frac{\partial A_y}{\partial y})^2 + (\frac{\partial A_y}{\partial z})^2 - (\frac{\partial A_z}{c\partial t})^2 + (\frac{\partial A_z}{\partial x})^2 + (\frac{\partial A_z}{\partial y})^2 + (\frac{\partial A_z}{\partial z})^2)$$

3. Apply:

$$-(\rho_q - \rho_m) = -\frac{\partial^2 \phi}{c\partial t^2} + c\frac{\partial^2 \phi}{\partial x^2} + c\frac{\partial^2 \phi}{\partial y^2} + c\frac{\partial^2 \phi}{\partial z^2}$$

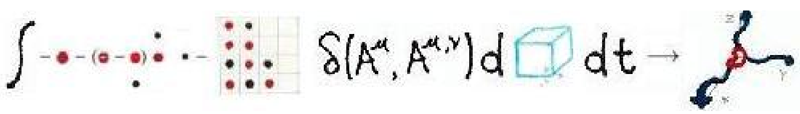
$$(\rho_q - \rho_m)\,\frac{\partial x}{c\partial t} = \frac{\partial^2 A_x}{c\partial t^2} - c\frac{\partial^2 A_x}{\partial x^2} - c\frac{\partial^2 A_x}{\partial y^2} - c\frac{\partial^2 A_x}{\partial z^2}$$

$$(\rho_q - \rho_m)\,\frac{\partial y}{c\partial t} = \frac{\partial^2 A_y}{c\partial t^2} - c\frac{\partial^2 A_y}{\partial x^2} - c\frac{\partial^2 A_y}{\partial y^2} - c\frac{\partial^2 A_y}{\partial z^2}$$

$$(\rho_q - \rho_m)\,\frac{\partial z}{c\partial t} = \frac{\partial^2 A_z}{c\partial t^2} - c\frac{\partial^2 A_z}{\partial x^2} - c\frac{\partial^2 A_z}{\partial y^2} - c\frac{\partial^2 A_z}{\partial z^2}$$

4. Executive summary:

$$J_q^{\nu} - J_m^{\nu} = \Box^2 A^{\nu}$$



Classical Fields

The classical fields $\vec{E}$ and $\vec{B}$ make up the antisymmetric tensor $(\nabla^{\mu}A^{\nu} - \nabla^{\mu}A^{\mu})$. Introduce three new fields, $\vec{e}$ and $\vec{b}$ which have EM counterparts, and a 4-vector field $\boldsymbol{g^{\mu}}$ for the diagonal components of the symmetric tensor $(\nabla^{\mu}A^{\nu} + \nabla^{\nu}A^{\mu})$.

- $\vec{E} = -\frac{\partial \vec{A}}{\partial t} - c\vec{\nabla}\phi$ Electric field.

- $\vec{e} = \frac{\partial \vec{A}}{\partial t} - c\vec{\nabla}\phi - 2\Gamma_{\sigma}{}^{0\,i}A^{\sigma}$ Symmetric analog to electric field.

- $\vec{B} = c\vec{\nabla} \times \vec{A}$ Magnetic field.

- $\vec{b} = -\partial^{i}A^{j} - \partial^{j}A^{i} - 2\Gamma_{\sigma}{}^{ij}A^{\sigma}$

  $= c(0, -\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} - 2\Gamma_{\sigma}{}^{yz}A^{\sigma}, -\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} - 2\Gamma_{\sigma}{}^{xz}A^{\sigma},$

  $-\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} - 2\Gamma_{\sigma}{}^{xy}A^{\sigma}) \equiv c\vec{\nabla} \boxtimes \vec{A}$

  Symmetric analog to magnetic field.

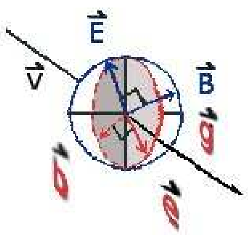
- $g^{\mu} = \partial^{\mu}A^{\mu} - \Gamma_{\sigma}{}^{\mu\mu}A^{\sigma}$

  $= (\frac{\partial \phi}{\partial t} - \Gamma_{\sigma}{}^{tt}A^{\sigma}, -c\frac{\partial A_x}{\partial x} - \Gamma_{\sigma}{}^{xx}A^{\sigma}, -c\frac{\partial A_y}{\partial y} - \Gamma_{\sigma}{}^{yy}A^{\sigma}, -c\frac{\partial A_z}{\partial z} - \Gamma_{\sigma}{}^{zz}A^{\sigma})$

  Diagonal of $\nabla^{\mu}A^{\nu}$.

3+3+3+3+4=16 fields total.

All three g's transform differently than axial or polar vectors.



Classical Fields in Detail

1. Start with the asymmetric field strength tensor, $\nabla^{\mu}A^{\nu}$, written as a matrix:

$$\begin{array}{cccc}
\mu=\phi & \mu=A_x & \mu=A_y & \mu=A_z \\
\frac{\partial\phi}{\partial t}-\Gamma_\sigma{}^{tt}A^\sigma & \frac{\partial A_x}{\partial t}-\Gamma_\sigma{}^{tx}A^\sigma & \frac{\partial A_y}{\partial t}-\Gamma_\sigma{}^{ty}A^\sigma & \frac{\partial A_z}{\partial t}-\Gamma_\sigma{}^{tz}A^\sigma \\
-c\frac{\partial\phi}{\partial x}-\Gamma_\sigma{}^{xt}A^\sigma & -c\frac{\partial A_x}{\partial x}-\Gamma_\sigma{}^{xx}A^\sigma & -c\frac{\partial A_y}{\partial x}-\Gamma_\sigma{}^{xy}A^\sigma & -c\frac{\partial A_z}{\partial x}-\Gamma_\sigma{}^{zz}A^\sigma \\
-c\frac{\partial\phi}{\partial y}-\Gamma_\sigma{}^{yt}A^\sigma & -c\frac{\partial A_x}{\partial y}-\Gamma_\sigma{}^{yx}A^\sigma & -c\frac{\partial A_y}{\partial y}-\Gamma_\sigma{}^{\mathrm{yy}}A^\sigma & -c\frac{\partial A_z}{\partial y}-\Gamma_\sigma{}^{yz}A^\sigma \\
-c\frac{\partial\phi}{\partial z}-\Gamma_\sigma{}^{zt}A^\sigma & -c\frac{\partial A_x}{\partial z}-\Gamma_\sigma{}^{zx}A^\sigma & -c\frac{\partial A_y}{\partial z}-\Gamma_\sigma{}^{zy}A^\sigma & -c\frac{\partial A_z}{\partial z}-\Gamma_\sigma{}^{zz}A^\sigma
\end{array}$$

2. An antisymmetric and symmetric sum equal to $2\nabla^\mu A^\nu$:

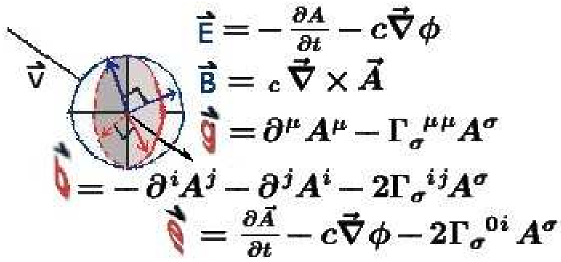
$$\nabla^\mu A^\nu-\nabla^\nu A^\mu = \begin{array}{cccc}
0 & \frac{\partial A_x}{\partial t}+c\frac{\partial\phi}{\partial x} & \frac{\partial A_y}{\partial t}+c\frac{\partial\phi}{\partial y} & \frac{\partial A_z}{\partial t}+\mathbf{c}\frac{\partial\phi}{\partial z} \\
-c\frac{\partial\phi}{\partial x}-\frac{\partial A_x}{\partial t} & 0 & -c\frac{\partial A_y}{\partial x}+c\frac{\partial A_x}{\partial y} & -c\frac{\partial A_z}{\partial x}+c\frac{\partial A_x}{\partial z} \\
-c\frac{\partial\phi}{\partial y}-\frac{\partial A_y}{\partial t} & -c\frac{\partial A_x}{\partial y}+c\frac{\partial A_y}{\partial x} & 0 & -c\frac{\partial A_z}{\partial y}+c\frac{\partial A_y}{\partial z} \\
-c\frac{\partial\phi}{\partial z}-\frac{\partial A_z}{\partial t} & -c\frac{\partial A_x}{\partial z}+c\frac{\partial A_z}{\partial x} & -c\frac{\partial A_y}{\partial z}+c\frac{\partial A_z}{\partial y} & 0
\end{array}$$

$$\nabla^\mu A^\nu+\nabla^\nu A^\mu =$$

$$\begin{array}{cccc}
2\frac{\partial\phi}{\partial t}-2\Gamma_\sigma{}^{tt}A^\sigma & \frac{\partial A_x}{\partial t}-c\frac{\partial\phi}{\partial x}-2\Gamma_\sigma{}^{tx}A^\sigma & \frac{\partial A_y}{\partial t}-c\frac{\partial\phi}{\partial y}-2\Gamma_\sigma{}^{ty}A^\sigma & -\frac{\partial A_z}{\partial t}-c\frac{\partial\phi}{\partial z}-2\Gamma_\sigma{}^{tz}A^\sigma \\
-c\frac{\partial\phi}{\partial x}+\frac{\partial A_x}{\partial t}-2\Gamma_\sigma{}^{xt}A^\sigma & 2c\frac{\partial A_x}{\partial x}-2\Gamma_\sigma{}^{xx}A^\sigma & -c\frac{\partial A_y}{\partial x}-c\frac{\partial A_x}{\partial y}-2\Gamma_\sigma{}^{xy}A^\sigma & -c\frac{\partial A_z}{\partial x}-c\frac{\partial A_x}{\partial z}-2\Gamma_\sigma{}^{xz}A^\sigma \\
-c\frac{\partial\phi}{\partial y}+\frac{\partial A_y}{\partial t}-2\Gamma_\sigma{}^{yt}A^\sigma & -c\frac{\partial A_x}{\partial y}-c\frac{\partial A_y}{\partial x}-2\Gamma_\sigma{}^{yx}A^\sigma & -2c\frac{\partial A_y}{\partial y}-2\Gamma_\sigma{}^{yy}A^\sigma & -c\frac{\partial A_z}{\partial y}-c\frac{\partial A_y}{\partial z}-2\Gamma_\sigma{}^{yz}A^\sigma \\
-c\frac{\partial\phi}{\partial z}+\frac{\partial A_z}{\partial t}-2\Gamma_\sigma{}^{zt}A^\sigma & -c\frac{\partial A_x}{\partial z}-c\frac{\partial A_z}{\partial x}-2\Gamma_\sigma{}^{zx}A^\sigma & -c\frac{\partial A_y}{\partial z}-c\frac{\partial A_z}{\partial y}-2\Gamma_\sigma{}^{zy}A^\sigma & -2c\frac{\partial A_z}{\partial z}-2\Gamma_\sigma{}^{zz}A^\sigma
\end{array}$$

3. $\nabla^\mu A^\nu$ written in terms of the gravitational, electric, and magnetic fields:

$$\begin{array}{cccc}
g_t & e_x-E_x & e_y-E_y & e_z-E_z \\
e_x+E_x & g_x & b_z-B_z & b_y+B_y \\
e_y+E_y & b_z+B_z & g_y & b_x-B_x \\
e_z+E_z & b_y-B_y & b_x+B_x & g_z
\end{array}$$



Gauss' Law and Newton's Gravitational Field

Method: $\frac{1}{2}$ (EM law + gravitational analog) + diagonal terms = field equations.

$$\rho_q-\rho_m=\frac{\partial^2\phi}{c\,\partial t^2}-c\frac{\partial^2\phi}{\partial x^2}-c\frac{\partial^2\phi}{\partial y^2}-c\frac{\partial^2\phi}{\partial z^2}$$

$$=\frac{\partial^2\phi}{c\,\partial t^2}+\frac{1}{2}\frac{\partial}{\partial x}\left(\left(-\frac{\partial A_x}{\partial t}-c\frac{\partial\phi}{\partial x}\right)+\left(\frac{\partial A_x}{\partial t}-c\frac{\partial\phi}{\partial x}\right)\right)$$

$$+\frac{1}{2}\frac{\partial}{\partial y}((-\frac{\partial A_y}{\partial t}-c\frac{\partial\phi}{\partial y})+(\frac{\partial A_y}{\partial t}-c\frac{\partial\phi}{\partial y}))$$

$$+\frac{1}{2}\frac{\partial}{\partial z}((-\frac{\partial A_z}{\partial t}-c\frac{\partial\phi}{\partial z})+(\frac{\partial A_z}{\partial t}-c\frac{\partial\phi}{\partial z}))$$

$$=\frac{\partial g_t}{c\partial t}+\frac{1}{2}(\vec{\nabla}\cdot\vec{E}+\vec{\nabla}\cdot\vec{e})$$

- Newton's [relativistic] gravitational field equation results in the physical situation where there is no electric charge density and no divergence of the field $\vec{E}$.

- Gauss' law results in the physical situation with no mass density and no divergence of the field $\vec{e}$.

Implications for forces: Newton's field law implies an attractive force for mass, while Gauss' law indicates like electric charges repulse.

Ampere's Law and Mass Current

Method: Same as previous.

$$\vec{J}_q-\vec{J}_m=(\frac{\partial^2 A_x}{c\,\partial t^2}-c\,\frac{\partial^2 A_x}{\partial x^2}-c\,\frac{\partial^2 A_x}{\partial y^2}-c\,\frac{\partial^2 A_x}{\partial z^2},\frac{\partial^2 A_y}{c\,\partial t^2}-c\,\frac{\partial^2 A_y}{\partial x^2}-c\,\frac{\partial^2 A_y}{\partial y^2}-c\,\frac{\partial^2 A_y}{\partial z^2},$$

$$\frac{\partial^2 A_z}{c\,\partial t^2}-c\,\frac{\partial^2 A_z}{\partial x^2}-c\,\frac{\partial^2 A_z}{\partial y^2}-c\,\frac{\partial^2 A_z}{\partial z^2})$$

$$=\frac{1}{2}(-\frac{\partial}{\partial t}((-\frac{\partial A_x}{c\,\partial t}-\frac{\partial\phi}{\partial x})-(\frac{\partial A_x}{c\,\partial t}-\frac{\partial\phi}{\partial x}))-\frac{\partial}{\partial x}\frac{\partial A_x}{\partial x}$$

$$-\frac{\partial}{\partial y}(\frac{\partial A_x}{\partial y}-\frac{\partial A_y}{\partial x})+\frac{\partial}{\partial z}(\frac{\partial A_z}{\partial x}-\frac{\partial A_x}{\partial z})+\frac{\partial}{\partial y}(-\frac{\partial A_x}{\partial y}-\frac{\partial A_y}{\partial x})+\frac{\partial}{\partial z}(-\frac{\partial A_z}{\partial x}-\frac{\partial A_x}{\partial z}),$$

$$-\frac{\partial}{\partial t}((-\frac{\partial A_y}{c\,\partial t}-\frac{\partial\phi}{\partial y})-(\frac{\partial A_y}{c\,\partial t}-\frac{\partial\phi}{\partial y}))-\frac{\partial}{\partial y}\frac{\partial A_y}{\partial y}$$

$$-\frac{\partial}{\partial z}(\frac{\partial A_y}{\partial z}-\frac{\partial A_z}{\partial y})+\frac{\partial}{\partial x}(\frac{\partial A_x}{\partial y}-\frac{\partial A_y}{\partial x})+\frac{\partial}{\partial z}(-\frac{\partial A_y}{\partial z}-\frac{\partial A_z}{\partial y})+\frac{\partial}{\partial x}(-\frac{\partial A_x}{\partial y}-\frac{\partial A_y}{\partial x}),$$

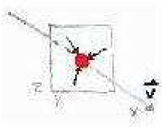
$$-\frac{\partial}{\partial t}((-\frac{\partial A_z}{c\,\partial t}-\frac{\partial\phi}{\partial z})-(\frac{\partial A_z}{c\,\partial t}-\frac{\partial\phi}{\partial z}))-\frac{\partial}{\partial z}\frac{\partial A_z}{\partial z}$$

$$-\frac{\partial}{\partial x}(\frac{\partial A_z}{\partial x}-\frac{\partial A_x}{\partial z})+\frac{\partial}{\partial y}(\frac{\partial A_y}{\partial z}-\frac{\partial A_z}{\partial y})+\frac{\partial}{\partial x}(-\frac{\partial A_z}{\partial x}-\frac{\partial A_x}{\partial z})+\frac{\partial}{\partial y}(-\frac{\partial A_y}{\partial z}-\frac{\partial A_z}{\partial y}))$$

$$=\frac{1}{2}(-\frac{\partial\vec{E}}{c\,\partial t}+\frac{\partial\,\vec{e}}{c\,\partial t}+\vec{\nabla}\times\vec{B}-\nabla\boxtimes\vec{b})+\vec{\nabla}^u g^u$$

- A pure mass current equation results in the physical situation where there is no electric current density no time change of the field $\vec{E}$ and no curl of the field $\vec{B}$.

- Ampere's law results in the physical situation where there is no mass current density, no gradient of the field $\boldsymbol{g^u}$ and not boxed curl of $\vec{\boldsymbol{b}}$.

Vector Identities

Vector identities or homogeneous equations are unchanged.

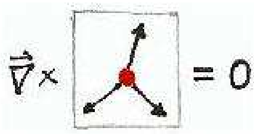
- No magnetic monopoles:

  $\vec{\nabla} \cdot \vec{B} = \vec{\nabla} \cdot (c\vec{\nabla} \times \vec{A}) = 0$

- Faraday's law:

  $\frac{\partial}{\partial t}\vec{\nabla} \times \vec{A} - \vec{\nabla} \times \frac{\partial \vec{A}}{\partial t} - \vec{\nabla} \times c\vec{\nabla}\phi = \vec{0}$

No obvious vector identity analogs for gravitational fields found yet.

Summary: Field Equations

Math:

$J^\nu_q - J^\nu_m = \Box^2 A^\nu$

Pictures:

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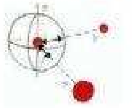
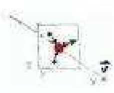
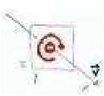
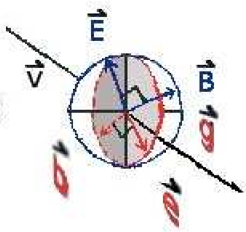
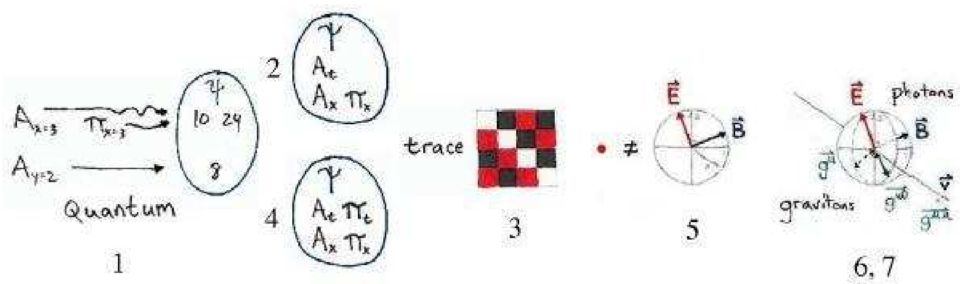
4 a  

4 b  

2, 3 

Quantization

1. Classical physics versus quantum mechanics.
2. Momentum from classic EM Lagrange density.
3. Quantizing EM fields by fixing the gauge.
4. Interpreting quantizing EM by fixing the Lorenz gauge.
5. Skeptical analysis of fixing the Lorenz gauge.
6. Momentum from GEM Lagrange density.
7. Interpreting GEM quantization.



Classical Physics versus Quantum Mechanics

Classical physics:

- Observables are numbers

  $A_x = 10 \quad A_y = 8 \quad \pi_x = 24$

- All observables are independent:

  $A_x\,\pi_x - \pi_x\,A_x = 0$

Quantum mechanics:

- Observables are operators that act on

  the wave function ψ to generate a number.

  $A_x|\psi\rangle = 10 \quad A_y|\psi\rangle = 8 \quad \pi_x|\psi\rangle = 24$

- Most observables are independent.

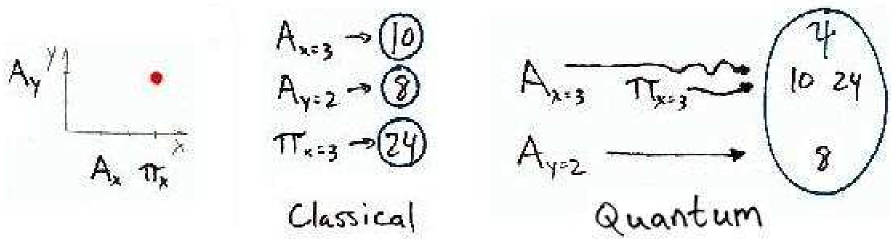
$[A_x, A_y]$ is called the *commutator*.

$A_x A_y|\psi\rangle - A_y A_x|\psi\rangle = [A_x, A_y]|\psi\rangle = 0$

- Conjugate observables are *not* independent.

  $[A_x, \pi_x]|\psi\rangle \neq 0$

Conjugate observables, like the potential and momentum, must have a non-zero commutator to quantize a field.



Momentum from Classic EM Lagrangian

1. Start with the EM Lagrange density written without indices.

   $$\mathfrak{L}_{\text{EM}} = -\frac{1}{\gamma}\rho_m - \frac{1}{c}J_\mu A^\mu - \frac{1}{4c^2}(\nabla^\mu A^\nu - \nabla^\nu A^\mu)(\nabla_\mu A_\nu - \nabla_\nu A_\mu)$$

   $$= -\rho_m(\sqrt{1 - (\frac{\partial x}{c\partial t})^2 - (\frac{\partial y}{c\partial t})^2 - (\frac{\partial z}{c\partial t})^2}$$

   $$-(\rho_q - \rho_m)(c\phi - \frac{\partial x}{\partial t}A_x - \frac{\partial x}{\partial t}A_y - \frac{\partial z}{\partial t}A_z)$$

   $$-\frac{1}{2}((\quad -(\frac{\partial \phi}{\partial x})^2 - (\frac{\partial \phi}{\partial y})^2 - (\frac{\partial \phi}{\partial z})^2$$

   $$-(\frac{\partial A_x}{c\partial t})^2 \qquad +(\frac{\partial A_x}{\partial y})^2 + (\frac{\partial A_x}{\partial z})^2$$

   $$-(\frac{\partial A_y}{c\partial t})^2 + (\frac{\partial A_y}{\partial x})^2 \qquad +(\frac{\partial A_y}{\partial z})^2$$

   $$-(\frac{\partial A_z}{c\partial t})^2 + (\frac{\partial A_z}{\partial x})^2 + (\frac{\partial A_z}{\partial y})^2$$

   $$-2\frac{\partial A_x}{c\partial t}\frac{\partial \phi}{\partial x} - 2\frac{\partial A_y}{c\partial t}\frac{\partial \phi}{\partial y} - 2\frac{\partial A_z}{c\partial t}\frac{\partial \phi}{\partial z}$$

   $$-2\frac{\partial A_y}{\partial z}\frac{\partial A_z}{\partial y} - 2\frac{\partial A_z}{\partial x}\frac{\partial A_x}{\partial z} - 2\frac{\partial A_x}{\partial y}\frac{\partial A_y}{\partial x})$$

2. Calculate momentum:

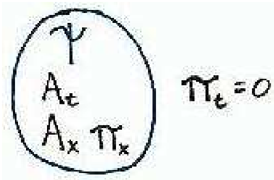
   $$\pi^\mu = h\sqrt{G}\,\frac{\partial \mathfrak{L}}{\partial(\frac{\partial A^\mu}{c\partial t})} = h\sqrt{G}\,(0, \frac{\partial A_x}{c\partial t} + \frac{\partial \phi}{\partial x}, \frac{\partial A_y}{c\partial t} + \frac{\partial \phi}{\partial y}, \frac{\partial A_z}{c\partial t} + \frac{\partial \phi}{\partial z})$$

Energy-momentum vector.

3. Momentum cannot be made into an operator:

   $[A_t, \pi_t]|\psi\rangle = [A_t, 0]|\psi\rangle = 0$

   Energy commutes with its conjugate operator.



Quantizing EM Fields by Fixing the Gauge

An EM gauge is a relationship between ϕ and $\vec{A}$ that does not change the Maxwell equations. Examples:

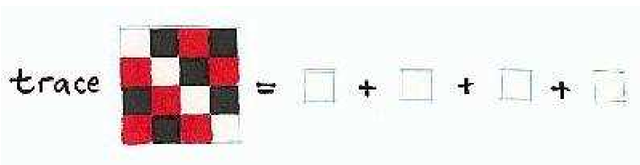
- Coulomb gauge.

  $\text{trace}(g_{\mu\nu}\nabla^\mu A^\nu) = \vec{\nabla} \cdot \vec{A} = 0$

- Lorenz gauge.

  $\text{trace}(g_{\mu\nu}\nabla^\mu A^\nu) = \frac{\partial\phi}{c\partial t} + \vec{\nabla} \cdot \vec{A} = 0$

For EM with no gravity, one is free to assign arbitrary values to the diagonal of the antisymmetric field strength tensor.



Quantizing EM by Fixing the Lorenz Gauge

Fix the Lorenz gauge in the EM Lagrange density.

1. Start with the Gupta-Bleuler Lagrange density written without indices:

$$\mathfrak{L}_{G-B} = -\frac{1}{\gamma}\rho_m - J_\mu A^\mu - \frac{1}{2c^2}(\nabla_\mu A^\mu)^2$$

$$-\frac{1}{4c^2}(\nabla^\mu A^\nu - \nabla^\nu A^\mu)(\nabla_\mu A_\nu - \nabla_\nu A_\mu)$$

$$= -\rho_m(\sqrt{1-(\frac{\partial x}{c\partial t})^2-(\frac{\partial y}{c\partial t})^2-(\frac{\partial z}{c\partial t})^2}$$

$$-(\rho_q - \rho_m)(c\phi - \frac{\partial x}{\partial t}A_x - \frac{\partial x}{\partial t}A_y - \frac{\partial z}{\partial t}A_z)$$

$$-\frac{1}{2}((\frac{\partial\phi}{c\partial t})^2 - (\frac{\partial\phi}{\partial x})^2 - (\frac{\partial\phi}{\partial y})^2 - (\frac{\partial\phi}{\partial z})^2$$

$$-(\frac{\partial A_x}{c\partial t})^2 + (\frac{\partial A_x}{\partial x})^2 + (\frac{\partial A_x}{\partial y})^2 + (\frac{\partial A_x}{\partial z})^2$$

$$-(\frac{\partial A_y}{c\partial t})^2 + (\frac{\partial A_y}{\partial x})^2 + (\frac{\partial A_y}{\partial y})^2 + (\frac{\partial A_y}{\partial z})^2$$

$$-(\frac{\partial A_z}{c\partial t})^2 + (\frac{\partial A_z}{\partial x})^2 + (\frac{\partial A_z}{\partial y})^2 + (\frac{\partial A_z}{\partial z})^2$$

$$-2\frac{\partial A_x}{c\partial t}\frac{\partial\phi}{\partial x} - 2\frac{\partial A_y}{c\partial t}\frac{\partial\phi}{\partial y} - 2\frac{\partial A_z}{c\partial t}\frac{\partial\phi}{\partial z}$$

$$-2\frac{\partial A_y}{\partial z}\frac{\partial A_z}{\partial y} - 2\frac{\partial A_z}{\partial x}\frac{\partial A_x}{\partial z} - 2\frac{\partial A_x}{\partial y}\frac{\partial A_y}{\partial x}$$

$$+2\frac{\partial\phi}{c\partial t}\frac{\partial A_x}{\partial x} + 2\frac{\partial\phi}{c\partial t}\frac{\partial A_y}{\partial y} + 2\frac{\partial\phi}{c\partial t}\frac{\partial A_z}{\partial z}$$

$$+2\frac{\partial A_x}{\partial x}\frac{\partial A_y}{\partial y} + 2\frac{\partial A_x}{\partial x}\frac{\partial A_z}{\partial z} + 2\frac{\partial A_y}{\partial y}\frac{\partial A_z}{\partial z})$$

2. Calculate momentum:

   $$\pi^\mu = h\sqrt{G}\,(-\frac{\partial\phi}{c\partial t} - \vec{\nabla}\cdot\vec{A}, \frac{\partial A_x}{c\partial t} + \frac{\partial\phi}{\partial x}, \frac{\partial A_y}{c\partial t} + \frac{\partial\phi}{\partial y}, \frac{\partial A_z}{c\partial t} + \frac{\partial\phi}{\partial z})$$

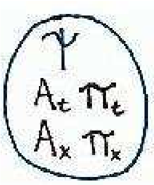
   Energy-momentum vector.

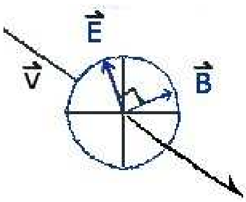
3. Momentum can be made into an operator:

Using the Euler-Lagrange equation [not shown], the equations of motion are identical to those of $\mathfrak{L}_{\mathrm{GEM}}$!

$J_q^\nu = \Box^2 A^\nu$

Reference: "Theory of longitudinal photons in quantum electrodynamics", Suraj N. Gupta, Proc. Phys. Soc. 63:681-691, 1950.

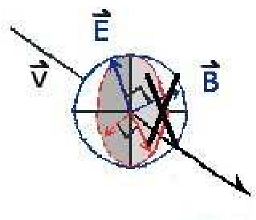




Interpreting the Gupta/Bleuler Quantization Method

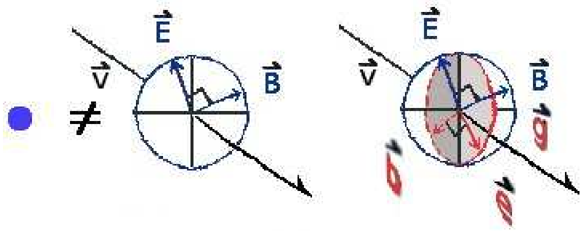
Results of quantization method:

- Four modes of transmission:
    1. Two transverse waves.
    2. One longitudinal wave.
    3. One scalar wave.
- Transverse waves are photons for EM.
- Scalar mode of transmission called a "scalar photon".
- "Supplementary condition" imposed to eliminate scalar and longitudinal photons as real particles, so they are always virtual.



Skeptical Analysis of Fixing the Lorenz Gauge

1. A scalar photon is an oxymoron. Photons must transform like vectors, even if photons happen to be virtual.
2. Eliminating an oxymoron cannot justify the supplementary condition.
3. A better interpretation for the 4D-wave equation of motion may exist.



Momentum from GEM Lagrange Density

1. Start with the GEM Lagrange density written without indices:

$$\mathfrak{L}_{\mathrm{GEM}} = -\frac{1}{\gamma}\rho_m - \frac{1}{c}(J^\mu_q - J^\mu_m)A_\mu - \frac{1}{2c^2}\nabla^\mu A^\nu \nabla_\mu A_\nu$$

$$= -\rho_m(\sqrt{1-(\frac{\partial x}{c\partial t})^2-(\frac{\partial y}{c\partial t})^2-(\frac{\partial z}{c\partial t})^2}$$

$$-(\rho_q-\rho_m)(c\phi - \frac{\partial x}{\partial t}A_x - \frac{\partial x}{\partial t}A_y - \frac{\partial z}{\partial t}A_z)$$

$$-\frac{1}{2}((\frac{\partial \phi}{c\partial t})^2 - (\frac{\partial \phi}{\partial x})^2 - (\frac{\partial \phi}{\partial y})^2 - (\frac{\partial \phi}{\partial z})^2$$

$$-(\frac{\partial A_x}{c\partial t})^2 + (\frac{\partial A_x}{\partial x})^2 + (\frac{\partial A_x}{\partial y})^2 + (\frac{\partial A_x}{\partial z})^2$$

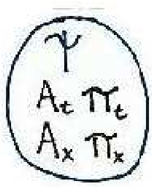
$$-(\frac{\partial A_y}{c\partial t})^2 + (\frac{\partial A_y}{\partial x})^2 + (\frac{\partial A_y}{\partial y})^2 + (\frac{\partial A_y}{\partial z})^2$$

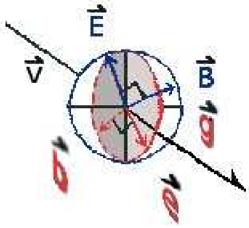
$$-(\frac{\partial A_z}{c\partial t})^2 + (\frac{\partial A_z}{\partial x})^2 + (\frac{\partial A_z}{\partial y})^2 + (\frac{\partial A_z}{\partial z})^2)$$

2. Calculate momentum:

$$\pi^\mu = h\sqrt{G}\,\frac{\partial \mathfrak{L}}{\partial(\frac{\partial A^\mu}{c\partial t})} = h\sqrt{G}\,(-\frac{\partial \phi}{c\partial t}, \frac{\partial A_x}{c\partial t}, \frac{\partial A_y}{c\partial t}, \frac{\partial A_z}{c\partial t})$$

3. Momentum can be made into an operator:





GEM Quantization

- Four modes of transmission:

   1. Two transverse waves.

2. One longitudinal wave.

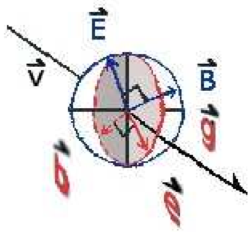
3. One scalar wave.

- Transverse waves are photons for EM.

- Longitudinal and scalar modes are gravitons of gravity traveling at the speed of light, generated by a symmetric rank-2 field strength tensor.

- General relativity predicts transverse waves, not scalar or longitudinal ones. The LIGO experiment to detect gravitational waves will be looking for transverse gravitational waves. GEM predicts the polarization will not be transverse.

- Gravitational modes are coupled to $\sqrt{G}$ and not h bar. This might get around negative energy problem because gravity quanta are not emitted.

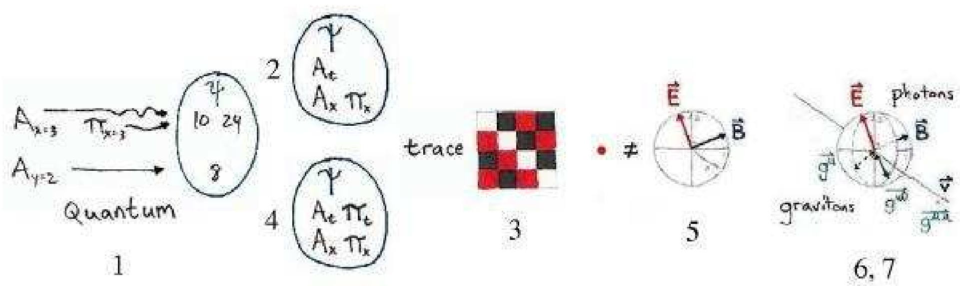


Summary: Quantization

Math:

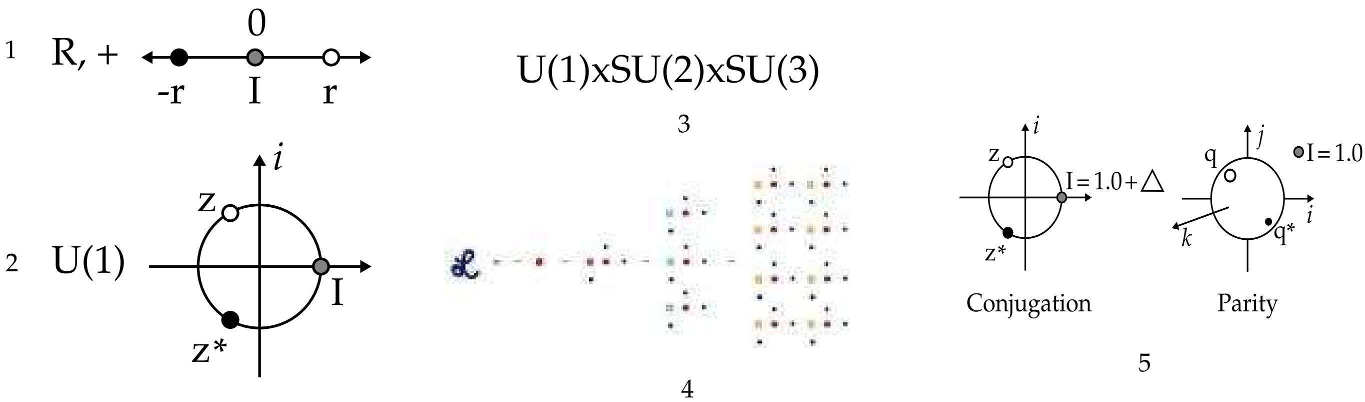
$$\pi^{\mu} = h\sqrt{G}\,\frac{\partial \mathfrak{L}}{\partial(\frac{\partial A^{\mu}}{c\partial t})} = h\sqrt{G}\,(-\frac{\partial \phi}{c\partial t}, \frac{\partial A_x}{c\partial t}, \frac{\partial A_y}{c\partial t}, \frac{\partial A_z}{c\partial t})$$

Pictures:



The Standard Model

1. Group theory.
2. Group theory by example.
3. The standard model.
4. The standard model Lagrange density.
5. Defining the multiplication operator.



Group Theory

Way to organize symmetry systematically.

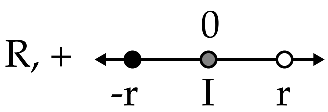
Definition: A set S with a binary operation ($\times$ or $+$)

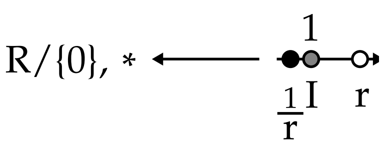
such that $s_1 \times s_2 \in S$ for all possible pairs of elements in S. A group has:

- An identity.
- An inverse for every element.
- Associative law holds.

Examples:

- Real numbers and $+$.
- Real numbers without 0 and $\times$.



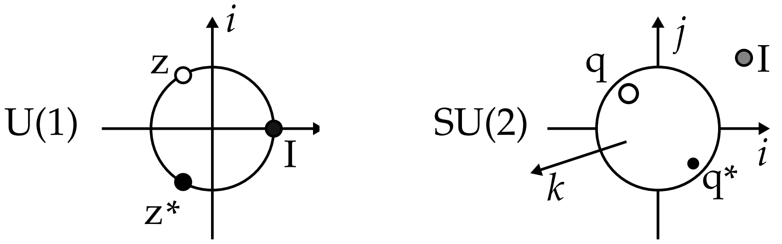


Group Theory by Example

- U(1), $z \times z^* = 1$, or unitary complex numbers.
  - $I = (1, 0)$ Identity is one.

- $z^{-1} = z^*$ Inverse is the conjugate.
- $z_1 \times z_2 = z_2 \times z_1$ Abelian.
- $U(\alpha) = e^{\begin{pmatrix} 0 & -\alpha \\ \alpha & 0 \end{pmatrix}}$ One number for the Lie algebra.

- SU(2), $q \times q^* = 1$, or unit quaternions (4D analog to complex numbers).
  - $I = (1, 0, 0, 0)$ Identity is one.
  - $q^{-1} = q^*$ Inverse is the conjugate.
  - $q_1 \times q_2 \neq q_2 \times q_1$ Non-Abelian.
  - $U(\alpha, \beta, \gamma) = e^{\begin{pmatrix} 0 & -\alpha & -\beta & -\gamma \\ \alpha & 0 & -\gamma & \beta \\ \beta & \gamma & 0 & -\alpha \\ \gamma & -\beta & \alpha & 0 \end{pmatrix}}$ Three numbers needed.



The Standard Model

Predicts patterns of all subatomic particles and three of four forces in Nature:

- U(1) Light, EM.
- SU(2) Weak force, radioactivity.
- SU(3) Strong force, the nucleus.

Says nothing about gravity.

U(1)xSU(2)xSU(3)

The Standard Model Lagrange Density

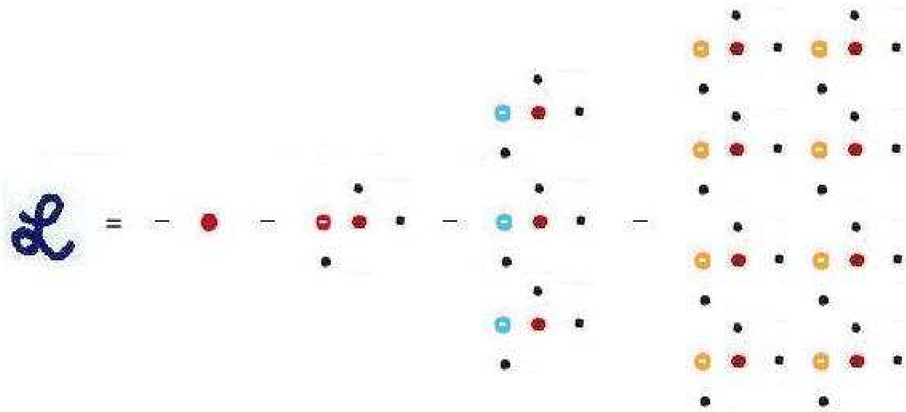
Describes all interactions of all subatomic forces in a volume.

$\mathfrak{L}_{\text{SM}} = \bar{\psi} \gamma^\mu D_\mu \psi$

$D_\mu = \partial_\mu - i g_{\text{EM}} Y A_\mu - i g_{\text{weak}} \frac{\tau^a}{2} W_\mu^a - i g_{\text{strong}} \frac{\lambda^b}{2} G_\mu^b$

- γ^μ Spinor matrix (no details provided here).

- $g_{...}$ Coupling constant to force.
- Y Generator of U(1) symmetry.
- $\tau^{a(1-3)}$ Generator of SU(2) symmetry.
- $\lambda^{b(1-8)}$ Generator of SU(3) symmetry.
- A_μ, W^a_μ, G^b_μ Complex-valued 4-potentials, two with internal symmetries.



Defining the Multiplication Operator

Given a pair of complex-valued 4-vectors,

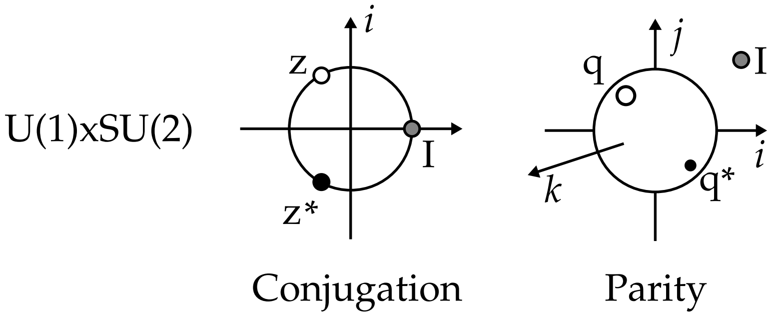
need to generate a real scalar.

Four components:

1. $(a, bi)^* = (a, -bi)$ Complex conjugation.
2. $(\phi, \vec{A})^p = (\phi, -\vec{A})$ Parity operator.
3. $g_{\mu\nu}$ Metric tensor.
4. $\frac{A^\mu}{|A|}$ Potentials normalized to themselves.

Define multiplication of 4-potentials in the standard model as:

$$\frac{A^\mu}{|A|}\frac{A^{\nu*p}}{|A|} g_{\mu\nu} = \frac{g_{tt}|A_t|^2 - g_{xx}|A_x|^2 - g_{yy}|A_y|^2 - g_{zz}|A_z|^2 - g_{\mu\nu}|A^\mu A^\nu|_{\mu\neq\nu}}{|A|^2}$$



Multiplication Operator in Spacetime

- $\frac{A^\mu}{|A|}\frac{A^{\nu*p}}{|A|}g_{\mu\nu}=1.0$ In flat spacetime.
- $\frac{A^\mu}{|A|}\frac{A^{\nu*p}}{|A|}g_{\mu\nu}=1.0+\delta$ In curved spacetime.

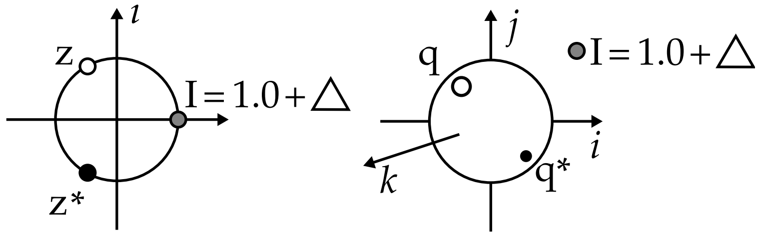
In curved spacetime, mass breaks U(1), SU(2), and SU(3)

symmetry in a precise way (circles get larger).

Y, τ^a, λ^b and the Higgs particle are not needed.

No new symmetry was added to the standard model. No new particle can be added. Instead, it may turn out that every particle can "act like a graviton" when it is involved with a distance measurement of the field.

U(1)xSU(2)



Conjugation Parity

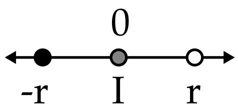
Summary: The Standard Model

Math:

$$\frac{A^\mu}{|A|}\frac{A^{\nu*p}}{|A|}g_{\mu\nu}=\frac{g_{\mathrm{tt}}|A_t|^2-g_{\mathrm{xx}}|A_x|^2-g_{\mathrm{yy}}|A_y|^2-g_{\mathrm{zz}}|A_z|^2-g_{\mu\nu}|A^\mu A^\nu|_{\mu\neq\nu}}{|A|^2}$$

Pictures:

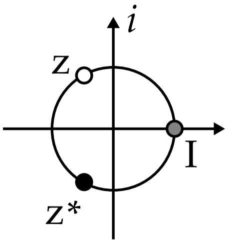
1 R, +

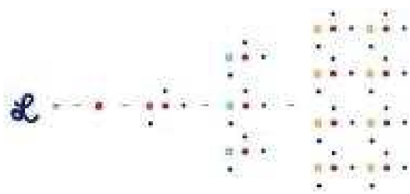


U(1)xSU(2)xSU(3)

3

2 U(1)





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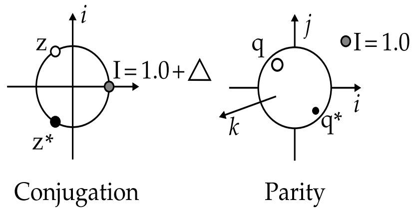


5

Must Do Physics Done for Day 2

1. $F_g = -Gm\psi\,\hat{R}$ Like charges attract.
2. $+m$ One charge.
3. $\rho = \nabla^2\phi$ Newton's gravitational field equation.
4. $m\frac{d^2\vec{R}}{dt^2} = -\frac{GMm}{R^2}\hat{R}$ Newton's law of gravity under classical conditions.
5. $d\tau^2 = (1 - 2\frac{GM}{c^2R} + 2(\frac{GM}{c^2R})^2)\,dt^2 - (1 + 2\frac{GM}{c^2R})\,\frac{dR^2}{c^2}$
   Consistent with the Schwarzschild metric.
6. $F_{\text{EM}} = q\,\vec{E}$ Like charges repel.
7. $\pm q$ Two distinct charges.
8. $\rho = \vec{\nabla}\cdot\vec{E} \quad \vec{J} = -\frac{\partial\vec{E}}{c\partial t} + \vec{\nabla}\times\vec{B}$ Maxwell source equations.
9. $0 = \vec{\nabla}\cdot\vec{B} \quad \vec{0} = \frac{\partial\vec{B}}{c\partial t} + \vec{\nabla}\times\vec{E}$ Maxwell homogeneous equations.
10. $F^\mu = q\,\frac{U_\nu}{c}(\nabla^\mu A^\nu - \nabla^\nu A^\mu)$ Lorentz force.
11. Unified field emission modes can be quantized.
12. Works with the standard model.
13. Indicates origin of mass.
14. LIGO (gravity wave polarization).
15. Rotation profiles of spiral galaxies.
16. Big Bang constant velocity distribution.

Caveats:

11. Operators not written explicitly. No calculations done.
12. SU(3) not investigated. Will spinors play nicely in framework?
13. Has the negative energy problem really been solved?

Summary Equations

1. GEM field equations as potentials:

   $J_q^\nu - J_m^\nu = \Box^2 A^\nu$

2. G fields

   a) $\vec{E}$ analog:

   $\vec{e} = \frac{\partial \vec{A}}{\partial t} - c\vec{\nabla}\phi$

   b) $\vec{B}$ analog:

   $\vec{b} = c\,\vec{\nabla} \boxtimes \vec{A} \equiv c(-\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}, -\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}, -\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y})$

   c) Diagonal:

   $g = \nabla^\mu A^\mu$

3. GEM field equations as classic fields

   a) GEM Gauss' law:

   $\rho_q - \rho_m = \frac{1}{2}(\vec{\nabla} \cdot \vec{E} + \vec{\nabla} \cdot \bar{e}) + \frac{\partial g_{00}}{c\partial t}$

   b) GEM Ampere's law:

   $\vec{J}_q - \vec{J}_m = \frac{1}{2}(-\frac{\partial \vec{E}}{c\,\partial t} + \frac{\partial\,\vec{e}}{c\,\partial t} + \vec{\nabla} \times \vec{B} - \vec{\nabla} \boxtimes \bar{b}) + \vec{\nabla}_u\,\overline{g^{\mathrm{uu}}}$

   c) No monopoles:

   $\vec{\nabla} \cdot \vec{B} = 0$

   d) Faraday's law:

   $\frac{\partial B}{c\partial t} + \vec{\nabla} \times \vec{E} = \vec{0}$

4. GEM momentum:

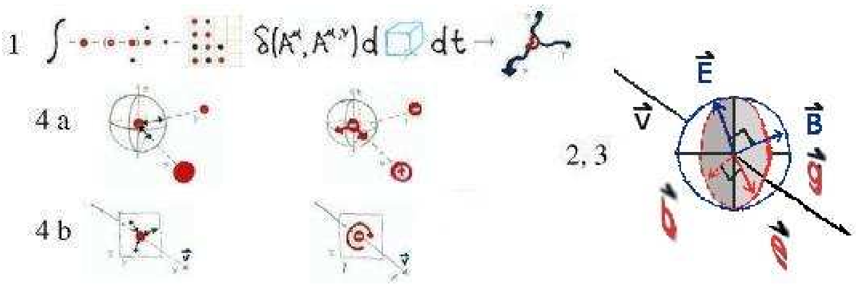
   $\pi^\mu = h\sqrt{G}\frac{\partial \mathfrak{L}}{\partial \frac{\partial A^\mu}{c\partial t}} = h\sqrt{G}\,(-\frac{\partial \phi}{c\partial t}, \frac{\partial A_x}{c\partial t}, \frac{\partial A_y}{c\partial t}, \frac{\partial A_z}{c\partial t})$

5. Standard model potential contraction:

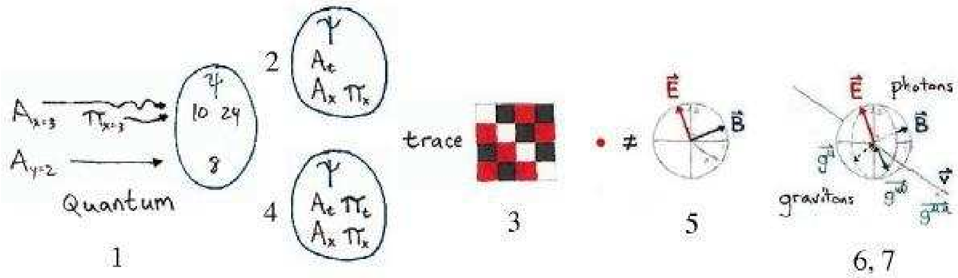
   $\frac{A^\mu}{|A|}\frac{A^{\nu*p}}{|A|}g_{\mu\nu} = 1$ For flat spacetime.

Summary Pictures

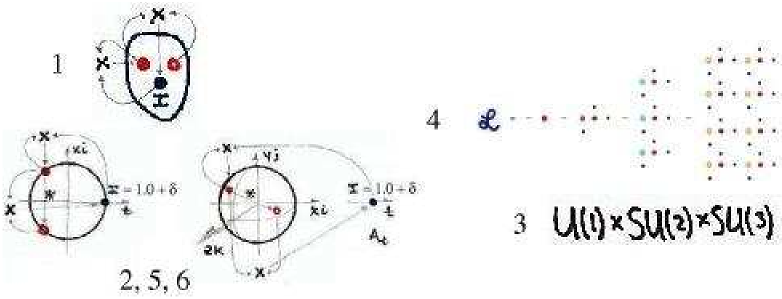
1. Fields:



2. Quantization:



3. The Standard Model:



4. Day 2: