Cosmological Consequences of the Chain Rule

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Abstract: The momentum density of a thin disc galaxy is composed of a constant velocity profile and a mass density that drops off exponentially with distance from the center. A relativistic force density obeys the chain rule:

$$\frac{d\rho V^\mu}{d\tau} = \rho \frac{dV^\mu}{d\tau} + V^\mu \frac{d\rho}{d\tau}$$

A derivative with respect to spacetime may in the classical limit become a derivative with respect to space or time. This classical gravitational law:

$$-\frac{GM\rho}{R^2}(\hat{R}+\hat{V}) = \rho \frac{V^2}{R}\hat{R} + \frac{d\rho}{d|R/c|}\vec{V}$$

appears consistent with outer velocity and mass density data for the galaxy NGC3198. The chain rule may eliminate the need for dark matter to explain motion of disc galaxies.

URL for slides http://theworld.com/sweetser/quaternions/talks/chain_rule/chain_rule.html

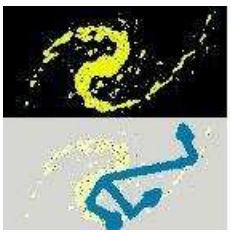
A Thin Disc Galaxy: NGC3198

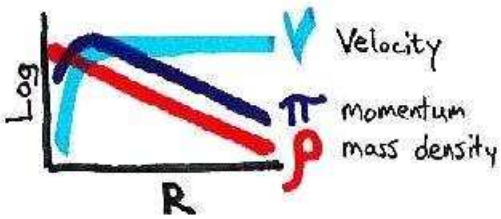
Not a point source, extending 200,000 light years across.

Outside the core region

- Velocity is **constant** at 150,000 m/s.
- Mass density decreases **exponentially**,

  mass/area = 37 Exp (-R'/2.23') solar masses/pc^2.
- Total mass is 1.0 x 10^{40} kg.





The BIG Newtonian Problems

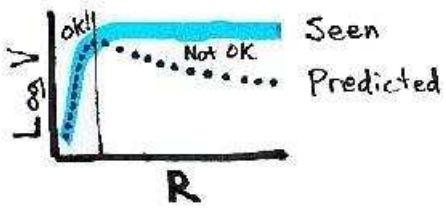
1. Unstable.

   Disc galaxies should have collapsed by now.

2. Velocity profile cannot remain flat.

   Newtonian constant V solution needs $M(R) = f(\frac{1}{R^2})$.

   An exponential drop at large radii is too fast for V to be constant.



Alternative hypotheses:

- Dark Matter.
- Modification of Newtonian Dynamics (MOND).

The Relativistic 4-Force

Since the specific Newtonian force law fails,

start from a more general 4-force law:

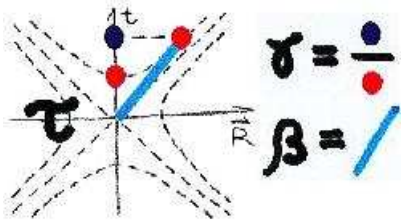
$F^{\mu} = \frac{d\rho V^{\mu}}{d\tau}$

- 4-velocity, V^{μ}.

  $V^{\mu} = c\,(\gamma, \gamma\beta)$

- Spacetime interval, τ.

  $(d\tau)^2 = (dt)^2 - (d\vec{R} \cdot d\vec{R})/c^2$



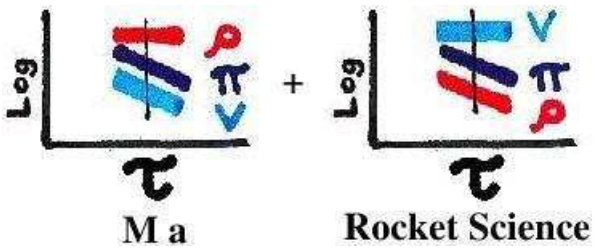
The Chain Rule for a 4-Force

The change in momentum with respect to spacetime equals

the change in 4-velocity with respect to the interval **plus**

the change in mass with respect to the spacetime interval.

$$\frac{d\rho V^{\mu}}{d\tau} = \rho \frac{dV^{\mu}}{d\tau} + V^{\mu} \frac{d\rho}{d\tau}$$



Relativistic to Classical Force

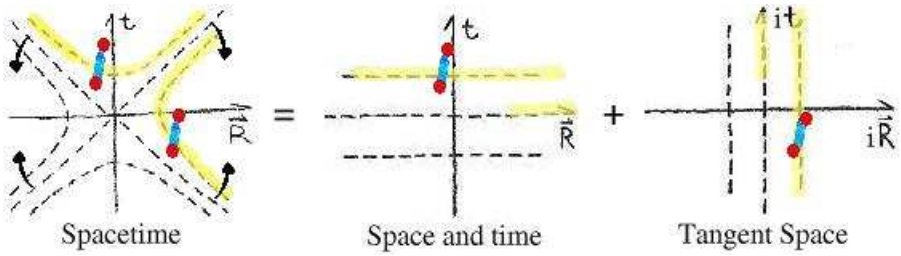
- 4-velocity becomes a 3-vector:

  $V^{\mu} \rightarrow \vec{V}$

- Change in **spacetime** becomes either

  $d\tau \rightarrow dt$, a change in **time** for timelike events in Newtonian space and time or

  $d\tau \rightarrow d|R/c|$, a change in **space** for spacelike events in the complex tangent space.



The **completely arbitrary** location of the spacetime origin means in the transformation from Minkowski spacetime to Newtonian space and time, the slope of some worldline for massive particles are real, while others are undefined in space and time unless the complex tangent space is included.

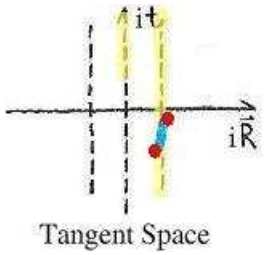
The Complex Tangent Space

The distance squared, $(d\tau)^2$, must be a real, positive definite number.

For spacelike separated events, $dR/c > dt$.

$$(d\tau)^2 = (idt)^2 - (idR/c)^2$$

is a real, positive definite number.



One cannot travel a distance iR, or measure a time it, however

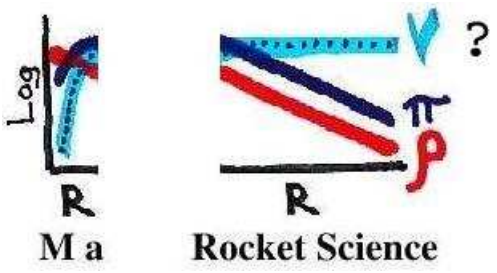
gamma, beta, and tau are all real numbers!

The Testable Hypothesis

The cause of gravity is the same, $-\frac{GM\rho}{R^2}$.

There is a new effect in the direction of the velocity vector in the complex tangent space.

$$\vec{F} = -\frac{GM\rho}{R^2}(\hat{R} + \hat{V}) = \rho\frac{V^2}{|R|}\hat{R} + \frac{d\rho}{d|R/c|}\vec{V}$$



Test of Hypothesis for Outer Profile

1. Start from Newtonian cause equal to rocket science effect:
   $-\frac{GM\rho}{R^2}\,\hat{V}\,?=?\,\frac{d\rho}{d|\vec{R}/c|}\,\vec{V}$

2. Collect on one side to form a first order differential equation:
   $(\frac{d\rho}{dR}+\frac{GM\rho}{cVR^2})\,\hat{V}\,=0$

3. Solve for the mass density:
   $\rho = k\,\mathrm{Exp}\,(\frac{GM}{cVR})$

4. Plug in values for the galaxy NGC3198:
   $G = 6.7x10^{-11}\,m^3\,\mathrm{kg}^{-1}\,s^{-2}, M = 1.0x10^{40}\,\mathrm{kg},$
   $c = 3.0x10^8\,m/s, V = 1.5x10^5\,m/s, 3.1x10^{16}\,m/'$
   $\rho = k\,\mathrm{Exp}\,(.48'/R')$

5. For large R:
   $\rho = k\,\mathrm{Exp}\,(.48'/R') \approx 1 + k\,\mathrm{Exp}\,(-R'/2.1')$

6. Note the similarity to the given mass density, ρ = 37 Exp (-R'/2.23').

Conclusion: The new "rocket science" effect of gravity deserves a detailed numerical study to see how well it agrees with all the data from the center outward.