Unifying Gravity and EM by Generalizing EM by sweetser@alum.mit.edu

The EM Lagrange density: $\mathfrak{L}_{\mathrm{EM}} = -\frac{1}{c} J_q^\mu A_\mu - \frac{1}{4c^2} (\nabla^\mu A^\nu - \nabla^\nu A^\mu)(\nabla_\mu A_\nu - \nabla_\nu A_\mu)$

Four generalizations:

- Like electric charges **repel** and like mass charges **attract**.
- **A**symmetric field strength tensor tensor,

  the sum of **anti**symmetric and **symmetric** tensors.
- **Exterior** and **covariant** derivatives.
- **Spin 1** and **spin 2** fields.

The GEM Lagrange density: $\mathfrak{L}_{\mathrm{GEM}} = -\frac{1}{c} (J_q^\mu - J_m^\mu) A_\mu - \frac{1}{2c^2} \nabla^\mu A^\nu \nabla_\mu A_\nu$

Apply the Euler-Lagrange equation to generate the field equations: $J_q^\mu - J_m^\mu = \Box^2 A^\mu$

Generalized Gauss' law: $\rho_q - \rho_m = \nabla_\nu \nabla^\nu \phi$

1. If $\rho_m = 0$, then $\rho_q = \nabla_\nu \nabla^\nu \phi$, which is Gauss' law.
2. If $\rho_q = 0$, $\frac{\partial^2 \phi}{c \partial t^2} = 0$, and $\Gamma = 0$, then $\rho_m = c \nabla^2 \phi$, which is Newton's law of gravity.
3. If $\rho_q = 0$ and $\partial_\nu \partial^\nu = 0$, then $\rho_m = \vec{\nabla} \cdot \Gamma_\sigma{}^{0i} A^\sigma$, ρ_m equals the divergence of the Christoffel symbol.

Exponential metric: $(\partial \tau)^2 = e^{-2 \frac{GM}{c^2 R}} (\partial t)^2 - e^{+2 \frac{GM}{c^2 R}} (\frac{\partial \vec{R}}{c})^2$

- Same 1st order PPN values as the Schwarzschild metric, so **passes** the same **tests**.
- Different 2nd order PPN values, so can be confirmed or rejected experimentally.
- Solves $\rho_m = \vec{\nabla} \cdot \Gamma_\sigma{}^{0i} A^\sigma$!

A 4D linear perturbation near 1/R that is electrically neutral is physically relevant:

diagonal SHO $A^\mu = \frac{c^2}{\sqrt{G}}$

$$\Big(\frac{1}{(\frac{1}{\sqrt{2}} + \frac{kx}{\sigma^2})^2 + (\frac{1}{\sqrt{2}} + \frac{ky}{\sigma^2})^2 + (\frac{1}{\sqrt{2}} + \frac{kz}{\sigma^2})^2 - (\frac{1}{\sqrt{2}} + \frac{kct}{\sigma^2})^2} + \frac{1}{(\frac{1}{\sqrt{2}} - \frac{kx}{\sigma^2})^2 + (\frac{1}{\sqrt{2}} - \frac{ky}{\sigma^2})^2 + (\frac{1}{\sqrt{2}} - \frac{kz}{\sigma^2})^2 - (\frac{1}{\sqrt{2}} + \frac{kct}{\sigma^2})^2},$$

$$\frac{1}{(\frac{1}{\sqrt{2}} + \frac{kx}{\sigma^2})^2 + (\frac{1}{\sqrt{2}} + \frac{ky}{\sigma^2})^2 + (\frac{1}{\sqrt{2}} + \frac{kz}{\sigma^2})^2 - (\frac{1}{\sqrt{2}} + \frac{kct}{\sigma^2})^2} + \frac{1}{(\frac{1}{\sqrt{2}} + \frac{kx}{\sigma^2})^2 + (\frac{1}{\sqrt{2}} - \frac{ky}{\sigma^2})^2 + (\frac{1}{\sqrt{2}} - \frac{kz}{\sigma^2})^2 - (\frac{1}{\sqrt{2}} - \frac{kct}{\sigma^2})^2},$$

$$\frac{1}{(\frac{1}{\sqrt{2}} + \frac{kx}{\sigma^2})^2 + (\frac{1}{\sqrt{2}} + \frac{ky}{\sigma^2})^2 + (\frac{1}{\sqrt{2}} + \frac{kz}{\sigma^2})^2 - (\frac{1}{\sqrt{2}} + \frac{kct}{\sigma^2})^2} + \frac{1}{(\frac{1}{\sqrt{2}} - \frac{kx}{\sigma^2})^2 + (\frac{1}{\sqrt{2}} + \frac{ky}{\sigma^2})^2 + (\frac{1}{\sqrt{2}} - \frac{kz}{\sigma^2})^2 - (\frac{1}{\sqrt{2}} - \frac{kct}{\sigma^2})^2},$$

$$\frac{1}{(\frac{1}{\sqrt{2}} + \frac{kx}{\sigma^2})^2 + (\frac{1}{\sqrt{2}} + \frac{ky}{\sigma^2})^2 + (\frac{1}{\sqrt{2}} + \frac{kz}{\sigma^2})^2 - (\frac{1}{\sqrt{2}} + \frac{kct}{\sigma^2})^2} + \frac{1}{(\frac{1}{\sqrt{2}} - \frac{kx}{\sigma^2})^2 + (\frac{1}{\sqrt{2}} - \frac{ky}{\sigma^2})^2 + (\frac{1}{\sqrt{2}} + \frac{kz}{\sigma^2})^2 - (\frac{1}{\sqrt{2}} - \frac{kct}{\sigma^2})^2}\Big)$$

The derivative has the correct, classical 1/distance^2 dependence to first order in k:

$\nabla^\mu A^\nu \cong \frac{c^2}{\sqrt{G}} \frac{k}{\sigma 2} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$ Can be used to derive the exponential metric.

A new class of constant velocity solutions for gravity:

$\vec{F} = -\frac{GM\rho}{R^2} (\hat{R} + \hat{V}) = \rho \frac{V^2}{|R|} \hat{R} + \frac{d\rho}{d|R/c|} \vec{V}$ For constant V: $m = k \,\mathrm{Exp}\, (\frac{GM}{cVR})$

Quantize with 2 spin fields: transverse modes for EM, scalar and longitudinal for gravity.

Conjugate momentum is non-zero: $\pi^\mu = h\sqrt{G} \frac{\partial \mathfrak{L}}{\partial \frac{\partial A^\mu}{c \partial t}} = h\sqrt{G} \, (-\frac{\partial \phi}{c \partial t}, \frac{\partial A_x}{c \partial t}, \frac{\partial A_y}{c \partial t}, \frac{\partial A_z}{c \partial t})$