4-Potential Equations for Gravity and Electromagnetism

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Abstract: To integrate classical and quantum field theory, 4-potential equations have worked for electromagnetism but not gravity. The following 4D wave equations embody the Maxwell equations and a relativistic field equation for gravity:

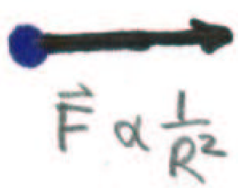
$$J_q^\mu - J_m^\mu = (\frac{\partial^2}{\partial t} - \nabla^2) A^\mu$$

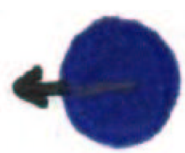
If the mass current density is zero, Maxwell equations in the Lorenz gauge result. If electric charge density and $\frac{\partial^2 \phi}{\partial t^2}$ are zero, then for $\mu = 0$, Newton's field equation for gravity results. A perturbation solution whose derivative has an inverse square distance dependence has been found. An equation consistent with special relativity can apply to classical gravity.

URL for slides: http://sdm.openacs.org/wp/presentation-top.tcl?presentation_id=1398

3 Problems with 4-Potential Gravity

1. Gravity attracts.
2. Force law depends on inverse square distance.
3. The equations are:
   - Rank 1,
   - Linear,
   - Bind to rest mass.





Gravity Attracts

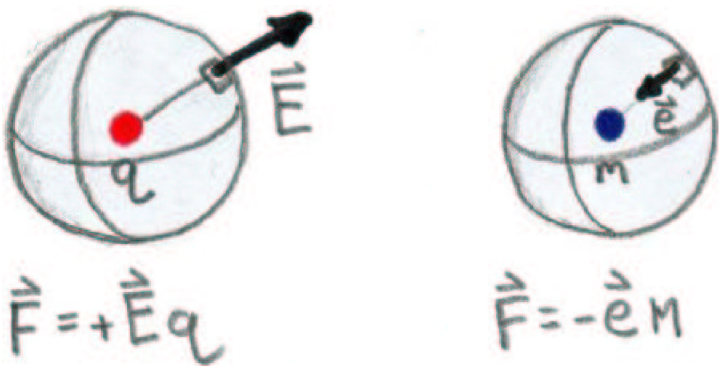
Problem: If gravity is modeled exactly on Maxwell, like charges repel.

Solution: One well-chosen minus sign.

$J_q^\mu - J_m^\mu$	$= (\frac{\partial^2}{\partial t^2} - \nabla^2) A^\mu$	Proposed unified field equations.
J_q^μ	$= (\frac{\partial^2}{\partial t^2} - \nabla^2) A^\mu$	Maxwell equations in the Lorenz gauge.
ρ_q	$= \quad - \nabla^2 \phi$	Gauss' (static) law, like charges **repel**.

$$-\rho_m = \quad -\nabla^2\phi$$ Newton's static field equations, like charges **attract**.

$$-\rho_m = (\frac{\partial^2}{\partial t^2} - \nabla^2)\,\phi$$ Relativistic gravitational field equation.



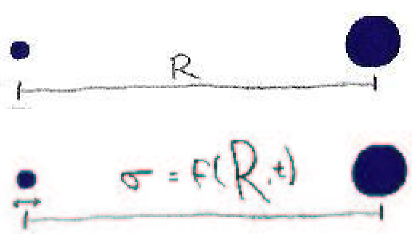
Inverse Squared Distance Force

Problem: Inverse distance square *potential* that solves 4D wave equation leads to an irrelevant inverse cube force law.

Solution: Perturbation near a physically relevent solution.

If static, then a potential solution is M/R, and the derivative is $-\frac{M}{R^2}$.

If not static but very nearly so, perturbation theory is required.



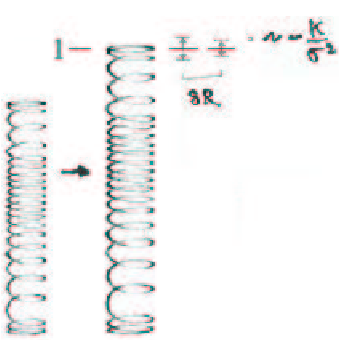
Perturbation Solution

Study linear perturbations of a normalized potential near $\sigma^2 = R^2 - c^2\,t^2$.

$$\phi = \frac{\phi'}{|\phi|} = \frac{\sigma^2}{\sigma'^2} = \frac{1}{(\frac{1}{\sqrt{2}} + \frac{k\,x}{\sigma^2})^2 + (\frac{1}{\sqrt{2}} + \frac{k\,y}{\sigma^2})^2 + (\frac{1}{\sqrt{2}} + \frac{k\,z}{\sigma^2})^2 - (\frac{1}{\sqrt{2}} + \frac{\mathrm{k}c\,t}{\sigma^2})^2}$$

If the spring constant k is small, then to first order in k:

$$\nabla\phi = -\frac{\sigma^2}{\sigma'^4}k\,\hat{R} + O(k^2) \cong -\frac{G\,M}{c^2\,R^2}\hat{R} \quad \text{if} \quad k = \frac{G\,M}{c^2}$$



Dogma Dogfight

General relativity dogma:

- Field equations are rank 2,
- Field equations are nonlinear,
- Gravity binds to energy-momentum.
- The Equivalence Principle:
  $m_p + \sum \eta^a E^a/c^2 = m_i + \sum \eta^a E^a/c^2$

Unified field dogma:

- Field equations are rank 1,
- Field equations are linear,
- Gravity binds to rest mass.
- The Equivalence Principle:
  $m_p = m_i$

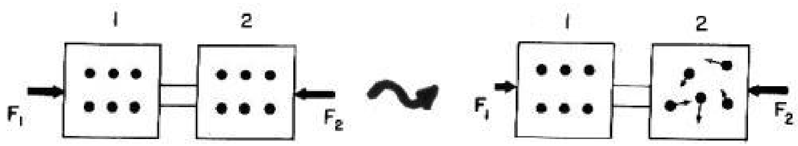
No data demonstrates the nonlinearity of gravity because gravity's effects are too weak.

$$G^{\mu\nu} = 8\pi T^{\mu\nu}$$

$$J_q^\mu - J_m^\mu = \Box^2 A^\mu$$

Thought Experiment Dilemma

Gravity binds to energy-momentum OR electric charge is conserved (not both).



All the rest mass of 1 particle becomes the kinetic energy of 5 others.

If gravity is a function of rest mass only, then the system accelerates without any external forces applied. Therefore gravity is a function of energy-momentum.

Make one specification: every particle is an electron, with 511 keV of rest masss and -1 unit of electric charge.

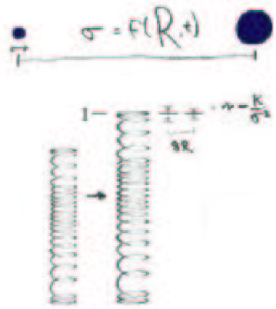
Box 2 has 511 keV of kinetic energy. Electric charge cannot be split into fifths. The box with 6 charges now has 5 moving charges. One unit of electric charge has been destroyed.

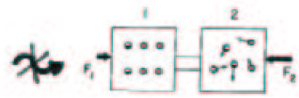
The data for electric charge conservation is exceptionally good.

The thought experiment is flawed, so gravity does not bind to energy-momentum.

Summary: 4-potential equations

Pictures:





Math:

$J_q^\mu - J_m^\mu = (\frac{\partial^2}{\partial t^2} - \nabla^2)\, A^\mu$

$\phi = \frac{1}{(\frac{1}{\sqrt{2}} + \frac{k\,x}{\sigma^2})^2 + (\frac{1}{\sqrt{2}} + \frac{k\,y}{\sigma^2})^2 + (\frac{1}{\sqrt{2}} + \frac{k\,z}{\sigma^2})^2 - (\frac{1}{\sqrt{2}} + \frac{\mathrm{kc}\,t}{\sigma^2})^2}$

$\nabla \phi \cong -\frac{G\,M}{c^2\,R^2}\,\hat{R}$