Unifying Gravity and EM, a Riddle You Can Solve

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Hilbert-Maxwell Action

$$S = \int \sqrt{-g}\, d^4x \left(R - \frac{1}{4}\left(\nabla^\mu A^\nu - \nabla^\nu A^\mu\right)\left(\nabla_\mu A_\nu - \nabla_\nu A_\mu\right)\right)$$

$$\delta g^{\mu\nu} \longrightarrow R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 0 \qquad \delta A^\mu \longrightarrow \Box^2 A^\mu = 0$$

Diff(M) **U(1)**

$$\nabla^\mu A^\nu - \nabla^\nu A^\mu = \partial^\mu A^\nu - \partial^\nu A^\mu$$

This is our best unification action for a vacuum to date, a simple combination of general relativity and Maxwell. The action has the Ricci scalar R for gravity, and the anti-symmetric tensor for EM. By varying the metric, Einstein's field equations are generated. By varying the potential, one sees the Maxwell equations in the Lorentz gauge. The covariant derivative is really an exterior derivative, one whose value in no way depends on the connection (assuming the connection is both torsion-free and metric compatible).

The action can also be characterized by groups. The group Diff(M) is all continuous changes in coordinates. Look at this group by example.

Sidebar

Diff(M) by Example

All smooth coordinate transformations

$$g_{\mu\nu} = \begin{bmatrix} 1 - 2\frac{GM}{c^2R} + 2(\frac{GM}{c^2R})^2 - ? & 0 & 0 & 0 \\ 0 & -1 - 2\frac{GM}{c^2R} - ? & 0 & 0 \\ 0 & 0 & -R^2 & 0 \\ 0 & 0 & 0 & -R^2\sin^2\theta \end{bmatrix}$$

Vacuum - Minkowski metric

0th order PPN metric - Classical Gravity

1st order PPN metric - Schwarzschild or Rosen

2nd order PPN metric - ??? - Will NOT be tested soon

Clifford Will, EGM8

Start with the Minkowski metric for a vacuum. Take steps away from this metric to ones related to gravity by adding additional terms. The first term added in red creates Newtonian gravity written in a metric form. The next two terms in blue form the first order Parameterized Post-Newtonian metrics that have been tested and confirmed by all weak field tests of gravity to date. The Schwarzschild metric has the same Taylor series expansion as the Rosen (or exponential) metric, so both metrics are indistinguishable at this level of measurement. At 2^{nd} order PPN accuracy, there will be a measurable difference of 0.7 microarcseconds of light bending around the Sun. According to Clifford Will, no experiment is in the planning stage to get the data for the 2^{nd} order PPN terms.

Hilbert-Maxwell GEM Action

$$S = \int \sqrt{-g}\, d^4x \left(R - \frac{1}{2}(\nabla^\mu A^\nu - \nabla^\nu A^\mu)(\nabla_\mu A_\nu - \nabla_\nu A_\mu)\right)$$

$$\delta g^{\mu\nu} \longrightarrow R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 0 \qquad \delta A^\mu \longrightarrow \Box^2 A^\mu = 0$$

Diff(M) ~ U(1)

$$\nabla^\mu A^\nu - \nabla^\nu A^\mu = \partial^\mu A^\nu - \partial^\nu A^\mu - \Gamma_\sigma^{\mu\nu} A^\sigma$$

The group U(1) is the unit circle in the complex plane. It arises from the antisymmetric tensor which forms a traceless matrix.

The Hilbert-Maxwell action has two problems. First it is not unified in a meaningful way. The varying the metric has nothing to do with the potential. Second, the equation cannot be quantized.

Create a new action by using less terms. The Ricci scalar R is a way to implement the group Diff(m) using a dynamic field. For the GEM action, the Diff(M) group is a symmetry of the action. The covariant derivative has two parts: the changes in the potential and the connection, a measure of how a symmetric tensor changes (assuming the connection is torsion-free and metric compatible). If one measures a covariant derivative, then one must choose how much is due to the changes in the ordinary derivative of the potential, and how much depends on the connection. In other words, there is a choice of gauge.

The U(1) symmetry is perfect for massless particles. If there is a mass, then the trace will not be zero. The size of the violation is beyond our ability to measure since the mass of an electron in units of electric charge is smaller than we have defined electric charge. The norm is no longer exactly 1.0, but is either very slightly larger or smaller than 1.0. This preserves the group structure necessary charge conservation. What changes is the diameter of the circle in the complex plane.

Encore: EGM8

General Gauss' Law

Solution 2: Constant potential with a exponential 1/R metric.

- $A^{\mu} = (a, b, c, d), \partial_{\mu} A^{\mu} = 0$
- $g_{\mu\nu} = \begin{pmatrix} e^{2(\frac{\sqrt{G}\, q - G m_p}{c^2 R})} & 0 & 0 & 0 \\ 0 & e^{-2(\frac{\sqrt{G}\, q - G m_p}{c^2 R})} & 0 & 0 \\ 0 & 0 & e^{-2(\frac{\sqrt{G}\, q - G m_p}{c^2 R})} & 0 \\ 0 & 0 & 0 & e^{-2(\frac{\sqrt{G}\, q - G m_p}{c^2 R})} \end{pmatrix}$
- $-\frac{1}{2} \nabla g^{00} \nabla g_{00} = -\nabla^2 \frac{q - \sqrt{G}\, m_p}{R} = (4.80x10^{-13} - 4.09x10^{-26}) C/m^3$

Because the metric is static and diagonal,
only the g_{00} term contributes to the divergence of the connection.

The field equations are the same as for EM. We know how to solve this equation for a flat, Euclidean metric where the connection is zero. For a static field, a charge/distance potential will work.

If one chooses a gauge where metric is constant, then a metric must be found that solves the field equations. The covariant differential operator could have a connection, so the goal is to find a metric whose divergence of its gradient is equal to zero..

Summary:
Start with the Hilbert-Maxwell action, and erase the Ricci scalar and the subtraction in the EM field strength tensor to make an asymmetric field strength tensor. U(1) symmetry is expanded to include potentials whose local norms ard not 1.0. The Diff(M) symmetry is preserved through the definition of a covariant derivative. A metric was found whose Laplacian solves the field equations. The 4D wave equation has been quantized in a manifestly covariant form before, but in the context of a spin 1 theory for EM exclusively. The same Gupta/Bleuler method can be used to quantize the modes of emission for the 4D wave equation, but this time there is a spin 2 field for gravity, so the technical problem of a negative probability density for that quantization approach disappears. Quantizing the GEM action leads to scalar and transverse modes of emission for the spin 2 field, in contrast to the transverse modes predicted by general relativity. Let us hope that both LIGO and LISA continue to get funding so that data can decide which theory remains viable.