Unifying Gravity and EM by Analogies to EM

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Abstract

Gravity and EM are unified at the level of a 4-potential by re-examining the most obvious choice at the Lagrangian level:

$$\mathfrak{L}_{\text{GEM}} = -\frac{\rho_m}{\gamma} - \frac{1}{c}\,(J_q^\mu - J_m^\mu)A_\mu - \frac{1}{2c^2}\,A^{\mu;\nu}A_{\mu;\nu}\,.$$

The Lagrangian has mass and electric current densities coupled to the 4-potential with different signs: an attractive one for gravity and one where like charges repel for EM. There are two irreducible field strength tensors that can represent the reducible asymmetric tensor $A^{\mu;\nu}$: a symmetric tensor for gravity and an antisymmetric electromagnetic field strength tensor $F^{\mu\nu}$ for EM. Any long-range interactions based on this Lagrangian will require two integer-spin fields due to the presence of two irreducible field strength tensors.

The Euler-Lagrange equations generate these 4D-wave field equations:

$$J_q^\mu - J_m^\mu = (\frac{\partial^2}{c\,\partial^2 t} - c\nabla^2)A^\mu.$$

If the mass current density J_m^μ is zero, the Maxwell equations in the Lorenz gauge result. If the electric charge density ρ_q is zero but the mass density ρ_m is not, in the static case, Newton's field equation for gravity results. For the non-static case, the equation is covariant under a Lorentz transformation.

An inverse distance function solves a 4D-wave equation. This appears to create a significant problem, since the derivative of the potential yields an inverse cubed force law which is not physical. The static $1/R$ potential function for Newton's field equation is physically relevant. If the system is no longer static, but only barely dynamic, then perturbation theory is necessary. A normalized, linear perturbation potential function was found whose derivative has the correct inverse distance dependence.

An electrically-neutral, normalized, linear 4-potential is used in a gravitational Lorentz 4-force equation. The solution to the force equation, with a flat spacetime metric constraint, yields the following dynamic metric equation:

$$(d\tau)^2 = e^{-2\frac{GM}{c^2 R}}(dt)^2 - e^{2\frac{GM}{c^2 R}}(dR/c)^2.$$

This metric equation has the same ten parameterized post-Newtonian (PPN) parameters as general relativity, so it will pass all the same tests of the equivalence principle and the Schwarzschild metric. The second-order PPN parameters are different, so the proposal could be confirmed or rejected experimentally.

A path from the gravitational Lorentz 4-force to Newton's classical 3-force is established. In the derivation, there is a choice between two derivatives as a consequence of applying the chain rule to momentum. The derivation is repeated, but this time choosing the other derivative. This leads to a new classical gravitational effect, one where the change in momentum caused by gravity has the effect that all particles travel with the same velocity, but the distribution of inertial mass changes with respect to distance. This may lead to new explanations for the rotation profile of thin galaxies without requiring dark matter or a modification of Newtonian mechanics. A new approach to the horizon and flatness problems of early big bang cosmology is possible. Both these application of the new classical gravitational effect will require mathematical modeling to see how well it fits the data.

A preliminary investigation into quantizing the radiation modes for the field theory is done. The process will be very similar to the Gupta-Bleuler method of fixing the Lorenz gauge due to the similarities between the field equations. The key difference is that the unified field equations have at least two spin fields, one even, the other odd.

A metric theory for gravity can only be about the distance between particles. A theory for gravity must play the same role within the context of the standard model. In flat spacetime, nothing about the standard model needs to be altered. In curved spacetime, mass changes the description of each group: the absolute value of the potentials in the U(1) group now are a definite value greater than 1.0, and the norm of the potentials in SU(2) and SU(3) are also greater than 1.0.

Chapter 1
Lagrange Densities

1.1 EM Lagrange Density

Where all EM energy is in a volume, no gravity.
$\mathfrak{L}_{\mathrm{EM}} = -\frac{\rho_m}{\gamma} - \frac{1}{c} J_q^\mu A_\mu - \frac{1}{4c^2}(A^{\mu.\nu} - A^{\nu,\mu})(A_{\mu,\nu} - A_{\nu,\mu})$

- $-\frac{\rho_m}{\gamma}$ Energy density of inertial mass in motion (KE).
- $-\frac{1}{c} J_q^\mu A_\mu$ Energy density of electric charge in motion (charge coupling).
- $-\frac{1}{4c^2}(A^{\mu.\nu} - A^{\nu,\mu})(A_{\mu,\nu} - A_{\nu,\mu})$

  Energy density of **anti**symmetric change in the potential.

1.2 EM to Gravity Analogy

- $-\frac{\rho_m}{\gamma}$ No change for inertial mass in motion (KE).
- $-q \longrightarrow +\sqrt{G}\, m$ Electric charge to mass charge (different coupling).
- Change field strength tensor's symmetry.
  1. $, \longrightarrow ;$ Derivatives to covariant derivatives.
  2. $A - A \longrightarrow A + A$ **Anti**symmetric to **sym**metric tensor.

There are two sign changes, both minus to plus. The first from -q to +m makes like mass charges attract in both the force and field equations. The second sign change in the tensor alters the symmetry, which also switches the kinds of particles that can carry out the interaction.

1.3 Gravity Lagrange Density Hypothesis

Where all gravitational energy is in a volume, no EM.

$$\mathfrak{L}_G = -\frac{\rho_m}{\gamma} + \frac{1}{c} J_m^\mu A_\mu - \frac{1}{4c^2}(A^{\mu;\nu} + A^{\nu;\mu})(A_{\mu;\nu} + A_{\nu;\mu})$$

- $-\frac{\rho_m}{\gamma}$ Energy density of inertial mass in motion (KE).
- $+\frac{1}{c} J_m^\mu A_\mu$ Energy density of mass charge in motion (charge coupling).
- $-\frac{1}{4c^2}(A^{\mu;\nu} + A^{\nu;\mu})(A_{\mu;\nu} + A_{\nu;\mu})$

  Energy density of **sym**metric change in the potential.

1.4 GEM Lagrange Density

$\mathfrak{L}_{\mathrm{GEM}}$ is the union of $\mathfrak{L}_G$ and $\mathfrak{L}_{\mathrm{EM}}$.

- Inertial mass in motion term is a union, not a sum.
- Sum charges in motion terms.
- Sum and simplify field strength tensor terms:
  1. $, \longrightarrow ;$ Derivatives to covariant derivatives.
  2. $A^{\mu;\nu}A_{\nu;\mu} - A^{\mu;\nu}A_{\nu;\mu} = 0$ Cross terms drop.
  3. $A^{\mu;\nu}A_{\mu;\nu} = A^{\nu;\mu}A_{\nu;\mu}$ Contractions are equal.

$$\mathfrak{L}_{\mathrm{GEM}} = -\frac{\rho_m}{\gamma} - \frac{1}{c}(J_q^\mu - J_m^\mu) A_\mu - \frac{1}{2\,c^2} A^{\mu;\nu}A_{\mu;\nu}$$

The asymmetric field strength tensor is composed of two irreducible field strength tensors. For long-range forces, the two tensors can be represented by different integral-spin fields, one even, one odd.

1.5 GEM Lagrange Density in Detail

Goal: Get to individual terms, no indices.

Method: Expand, contract, and repeat.

1. Start with the GEM Lagrange density which has $1+4+16$ final terms:
   $$\mathfrak{L}_{\mathrm{GEM}} = -\frac{\rho_m}{\gamma} - \frac{1}{c}(J_q^\mu - J_m^\mu)A_\mu - \frac{1}{2c^2} A^{\mu;\nu}A_{\mu;\nu}$$
2. Expand J^μ and A_μ. Apply the definition of a contravariant derivative to a contravariant vector $(A^{\mu;\nu} = A^{\mu,\nu} - \Gamma_{\varpi}{}^{\mu\nu} A^{\varpi})$ to $A^{\mu;\nu}$ and $A_{\mu;\nu}$:
   $$\mathfrak{L} = -\frac{\rho_m}{\gamma} - \frac{1}{c}(\rho_q - \rho_m, J_q^u - J_q^u)(\phi, -A^u)$$
   $$-\frac{1}{2c^2}(A^{\mu,\nu} - \Gamma_{\varpi}{}^{\mu\nu}A^{\varpi})(A_{\mu,\nu} - \Gamma^{\sigma}{}_{\mu\nu}A_{\sigma})$$
3. Contract J with A. Multiply out final term:

$$\mathfrak{L} = -\frac{\rho_m}{\gamma} - \frac{(\rho_q - \rho_m)}{c^2}(c\phi - v^u A^u)$$
$$- \frac{1}{2c^2}(A^{\mu,\nu}A_{\mu,\nu} - 2\Gamma_{\varpi}{}^{\mu\nu}A^{\varpi}A_{\mu,\nu} - \Gamma_{\varpi}{}^{\mu\nu}A^{\varpi}\Gamma^{\sigma}{}_{\mu\nu}A_{\sigma})$$

4. Expand $A^{\mu,\nu}$ and $A_{\mu,\nu}$. Work in local covariant coordinates where $\Gamma = 0$:

$$\mathfrak{L} = -\frac{\rho_m}{\gamma} - \frac{(\rho_q - \rho_m)}{c^2}(c\phi - v^u A^u) - \frac{1}{2c^2}(\frac{\partial}{\partial t}, -\nabla^v)(\phi, A^u)(\frac{\partial}{\partial t}, \nabla^v)(\phi, -A^u)$$

5. Contract the second rank tensors, using the lines above as a visual guide:

$$\mathfrak{L} = -\frac{\rho_m}{\gamma} - \frac{(\rho_q - \rho_m)}{c^2}(c\phi - v^u A^u) - \frac{1}{2c^2}((\frac{\partial\phi}{\partial t})^2 - (\nabla\phi)^{v2} - (\frac{\partial A}{\partial t})^{u2} + (\nabla A)^{uv2}$$

6. Write it ALL out:

$$\mathfrak{L} = -\rho_m\sqrt{1 - (\frac{\partial x}{c\partial t})^2 - (\frac{\partial y}{c\partial t})^2 - (\frac{\partial z}{c\partial t})^2}$$
$$- \frac{(\rho_q - \rho_m)}{c^2}(c\phi - \frac{\partial x}{\partial t}A_x - \frac{\partial x}{\partial t}A_y - \frac{\partial z}{\partial t}A_z)$$
$$- \frac{1}{2}((\frac{\partial\phi}{c\partial t})^2 - (\frac{\partial\phi}{\partial x})^2 - (\frac{\partial\phi}{\partial y})^2 - (\frac{\partial\phi}{\partial z})^2 - (\frac{\partial A_x}{c\partial t})^2 + (\frac{\partial A_x}{\partial x})^2 + (\frac{\partial A_x}{\partial y})^2 + (\frac{\partial A_x}{\partial z})^2$$
$$- (\frac{\partial A_y}{c\partial t})^2 + (\frac{\partial A_y}{\partial x})^2 + (\frac{\partial A_y}{\partial y})^2 + (\frac{\partial A_y}{\partial z})^2 - (\frac{\partial A_z}{c\partial t})^2 + (\frac{\partial A_z}{\partial x})^2 + (\frac{\partial A_z}{\partial y})^2 + (\frac{\partial A_z}{\partial z})^2$$

1.6 In Pictures

The EM Lagrange density: $-$ KE $-$ electric $J.A$ $-$ antisymmetric field.field

The gravity Lagrange density: $-$ KE $+$ mass $J.A$ $-$ symmetric field.field

The GEM Lagrange density: $-$ KE $-$ (electric $-$ mass) $J.A$ $-$ asymmetric field.field

Chapter 2
Fields

2.1 The Players

Here is a table of the players in the field equations. Three new fields for gravity will be defined subsequently.

Rank	*Symbol*	*Name*
0	$\mathfrak{L}$	Lagrange density
1	$c\frac{\partial \mathfrak{L}}{\partial A^{\mu}} = c(\frac{\partial \mathfrak{L}}{\partial A^{\mu,\nu}})^{,\nu}$	Field equations
1	A^{μ}	4-Potential
2	$A^{\mu,\nu}$	4-Derivative of a 4-potential
2	$\vec{E}\,, \vec{e}\,, \vec{B}\,, \vec{b}\,, g^{\mu}$	Classical fields which constitute $A^{\mu,\nu}$
1	$\frac{\partial \vec{E}}{c\partial t}, \vec{\nabla} \times \vec{E}\,, \frac{\partial\, \vec{e}}{c\partial t}, \frac{\partial \vec{B}}{c\partial t}, \vec{\nabla} \times \vec{B}\,, \vec{\nabla} \boxtimes \vec{b}\,, \nabla g^{\mu}$	Field equations terms written as classical fields

2.2 Apply Euler-Lagrange to the GEM Lagrangian

1. Start with the Euler-Lagrange equation, $\frac{\partial \mathfrak{L}}{\partial A^{\mu}} = (\frac{\partial \mathfrak{L}}{\partial A^{\mu,\nu}})^{,\nu}$, written without indices:
$$c\frac{\partial \mathfrak{L}}{\partial \phi} = c(\frac{\partial}{\partial t}(\frac{\partial \mathfrak{L}}{\partial(\frac{\partial \phi}{\partial t})}) - \frac{\partial}{\partial x}(\frac{\partial \mathfrak{L}}{\partial(-\frac{\partial \phi}{\partial x})}) - \frac{\partial}{\partial y}(\frac{\partial \mathfrak{L}}{\partial(-\frac{\partial \phi}{\partial y})}) - \frac{\partial}{\partial z}(\frac{\partial \mathfrak{L}}{\partial(-\frac{\partial \phi}{\partial z})}))$$
$$c\frac{\partial \mathfrak{L}}{\partial A_x} = c(\frac{\partial}{\partial t}(\frac{\partial \mathfrak{L}}{\partial(\frac{\partial A_x}{\partial t})}) - \frac{\partial}{\partial x}(\frac{\partial \mathfrak{L}}{\partial(-\frac{\partial A_x}{\partial x})}) - \frac{\partial}{\partial y}(\frac{\partial \mathfrak{L}}{\partial(-\frac{\partial A_x}{\partial y})}) - \frac{\partial}{\partial z}(\frac{\partial \mathfrak{L}}{\partial(-\frac{\partial A_x}{\partial z})}))$$
$$c\frac{\partial \mathfrak{L}}{\partial A_y} = c(\frac{\partial}{\partial t}(\frac{\partial \mathfrak{L}}{\partial(\frac{\partial A_y}{\partial t})}) - \frac{\partial}{\partial x}(\frac{\partial \mathfrak{L}}{\partial(-\frac{\partial A_y}{\partial x})}) - \frac{\partial}{\partial y}(\frac{\partial \mathfrak{L}}{\partial(-\frac{\partial \mathrm{Ay}}{\partial y})}) - \frac{\partial}{\partial z}(\frac{\partial \mathfrak{L}}{\partial(-\frac{\partial A_y}{\partial z})}))$$
$$c\frac{\partial \mathfrak{L}}{\partial A_z} = c(\frac{\partial}{\partial t}(\frac{\partial \mathfrak{L}}{\partial(\frac{\partial A_z}{\partial t})}) - \frac{\partial}{\partial x}(\frac{\partial \mathfrak{L}}{\partial(-\frac{\partial A_z}{\partial x})}) - \frac{\partial}{\partial y}(\frac{\partial \mathfrak{L}}{\partial(-\frac{\partial \mathrm{Az}}{\partial y})}) - \frac{\partial}{\partial z}(\frac{\partial \mathfrak{L}}{\partial(-\frac{\partial A_z}{\partial z})}))$$

2. Write out GEM Lagrange density without indices:
$$\mathfrak{L} = -\rho_m(\sqrt{1 - (\frac{\partial x}{c\partial t})^2 - (\frac{\partial y}{c\partial t})^2 - (\frac{\partial z}{c\partial t})^2}$$
$$-\frac{(\rho_q - \rho_m)}{c^2}(c\phi - \frac{\partial x}{\partial t}A_x - \frac{\partial x}{\partial t}A_y - \frac{\partial z}{\partial t}A_z)$$
$$-\frac{1}{2}((\frac{\partial \phi}{c\partial t})^2 - (\frac{\partial \phi}{\partial x})^2 - (\frac{\partial \phi}{\partial y})^2 - (\frac{\partial \phi}{\partial z})^2 - (\frac{\partial A_x}{c\partial t})^2 + (\frac{\partial A_x}{\partial x})^2 + (\frac{\partial A_x}{\partial y})^2 + (\frac{\partial A_x}{\partial z})^2$$
$$-(\frac{\partial A_y}{c\partial t})^2 + (\frac{\partial A_y}{\partial x})^2 + (\frac{\partial A_y}{\partial y})^2 + (\frac{\partial A_y}{\partial z})^2 - (\frac{\partial A_z}{c\partial t})^2 + (\frac{\partial A_z}{\partial x})^2 + (\frac{\partial A_z}{\partial y})^2 + (\frac{\partial A_z}{\partial z})^2$$

3. Apply:
$$-(\rho_q - \rho_m) = -\frac{\partial^2 \phi}{c\partial t^2} + c\frac{\partial^2 \phi}{\partial x^2} + c\frac{\partial^2 \phi}{\partial y^2} + c\frac{\partial^2 \phi}{\partial z^2}$$
$$(\rho_q - \rho_m)\,\frac{\partial x}{c\partial t} = \frac{\partial^2 A_x}{c\partial t^2} - c\frac{\partial^2 A_x}{\partial x^2} - c\frac{\partial^2 A_x}{\partial y^2} - c\frac{\partial^2 A_x}{\partial z^2}$$
$$(\rho_q - \rho_m)\,\frac{\partial y}{c\partial t} = \frac{\partial^2 A_y}{c\partial t^2} - c\frac{\partial^2 A_y}{\partial x^2} - c\frac{\partial^2 A_y}{\partial y^2} - c\frac{\partial^2 A_y}{\partial z^2}$$
$$(\rho_q - \rho_m)\,\frac{\partial z}{c\partial t} = \frac{\partial^2 A_z}{c\partial t^2} - c\frac{\partial^2 A_z}{\partial x^2} - c\frac{\partial^2 A_z}{\partial y^2} - c\frac{\partial^2 A_z}{\partial z^2}$$

4. Summary:
$$J_q^\mu - J_m^\mu = \left(\frac{\partial^2}{c\,\partial t^2} - c\nabla^2\right) A^\mu$$

- If J_m^μ is zero, the Maxwell equations in the Lorenz gauge result.
- If ρ_q is zero, for a static field, Newton's gravitational field law results.
- If ρ_q is zero, for a dynamic field, a relativistic gravitational field results.

2.3 Classical Fields

The classical fields $\vec{E}$ and $\vec{B}$ together make up the antisymmetric tensor $(A^{\mu,\nu} - A^{\nu,\mu})$. Introduce three new fields, $\vec{e}$ and $\vec{b}$ which have EM counterparts, and a 4-vector field g^μ for the diagonal components of the symmetric tensor $(A^{\mu,\nu} + A^{\nu,\mu})$.

- $\vec{E} = -\frac{\partial \vec{A}}{\partial t} - c\vec{\nabla}\phi$ Electric field.
- $\vec{e} = +\frac{\partial \vec{A}}{\partial t} - c\vec{\nabla}\phi$ Symmetric analog to electric field.
- $\vec{B} = c\vec{\nabla} \times \vec{A}$ Magnetic field.
- $\vec{b} = c(0, -\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}, -\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}, -\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}) \equiv c\vec{\nabla} \boxtimes \vec{A}$
  Symmetric analog to magnetic field.
- $g^\mu = A^{\mu,\mu} = (\frac{\partial \phi}{\partial t}, -c\frac{\partial A_x}{\partial x}, -c\frac{\partial A_y}{\partial y}, -c\frac{\partial A_z}{\partial z})$ Diagonal of $A^{\mu,\nu}$.

3+3+3+3+4=16 parts total. All three new fields transform differently from axial or polar vectors.

2.4 Classical Fields in Detail

1. Start with the asymmetric unified field strength tensor, $A^{\mu,\nu}$, written as a matrix:

$$A^{\mu,\nu} = \begin{pmatrix} \frac{\partial}{\partial t} \\ -c\frac{\partial}{\partial x} \\ -c\frac{\partial}{\partial y} \\ -c\frac{\partial}{\partial z} \end{pmatrix} \begin{pmatrix} \phi & A_x & A_y & A_z \end{pmatrix} = \begin{pmatrix} \frac{\partial \phi}{\partial t} & \frac{\partial A_x}{\partial t} & \frac{\partial A_y}{\partial t} & \frac{\partial A_z}{\partial t} \\ -c\frac{\partial \phi}{\partial x} & -c\frac{\partial A_x}{\partial x} & -c\frac{\partial A_y}{\partial x} & -c\frac{\partial A_z}{\partial x} \\ -c\frac{\partial \phi}{\partial y} & -c\frac{\partial A_x}{\partial y} & -c\frac{\partial A_y}{\partial y} & -c\frac{\partial A_z}{\partial y} \\ -c\frac{\partial \phi}{\partial z} & -c\frac{\partial A_x}{\partial z} & -c\frac{\partial A_y}{\partial z} & -c\frac{\partial A_z}{\partial z} \end{pmatrix}$$

2. An antisymmetric and symmetric tensor sum equals $2A^{\mu,\nu}$:

$$A^{\mu,\nu} - A^{\nu,\mu} = \begin{pmatrix} 0 & \frac{\partial A_x}{\partial t} + c\frac{\partial \phi}{\partial x} & \frac{\partial A_y}{\partial t} + c\frac{\partial \phi}{\partial y} & \frac{\partial A_z}{\partial t} + c\frac{\partial \phi}{\partial z} \\ -c\frac{\partial \phi}{\partial x} - \frac{\partial A_x}{\partial t} & 0 & -c\frac{\partial A_y}{\partial x} + \frac{\partial A_y}{\partial x} & -c\frac{\partial A_z}{\partial x} + c\frac{\partial A_x}{\partial z} \\ -c\frac{\partial \phi}{\partial y} - \frac{\partial A_y}{\partial t} & -c\frac{\partial A_x}{\partial y} + c\frac{\partial A_y}{\partial x} & 0 & -c\frac{\partial A_z}{\partial y} + c\frac{\partial A_y}{\partial z} \\ -c\frac{\partial \phi}{\partial z} - \frac{\partial A_z}{\partial t} & -c\frac{\partial A_x}{\partial z} + c\frac{\partial A_z}{\partial x} & -c\frac{\partial A_y}{\partial z} + c\frac{\partial A_z}{\partial y} & 0 \end{pmatrix}$$

$$A^{\mu,\nu} + A^{\nu,\mu} = \begin{pmatrix} 2\frac{\partial \phi}{\partial t} & \frac{\partial A_x}{\partial t} - c\frac{\partial \phi}{\partial x} & \frac{\partial A_y}{\partial t} - c\frac{\partial \phi}{\partial y} & \frac{\partial A_z}{\partial t} - c\frac{\partial \phi}{\partial z} \\ -c\frac{\partial \phi}{\partial x} + \frac{\partial A_x}{\partial t} & -2c\frac{\partial A_x}{\partial x} & -c\frac{\partial A_y}{\partial x} - \frac{\partial A_y}{\partial x} & -c\frac{\partial A_z}{\partial x} - c\frac{\partial A_x}{\partial z} \\ -c\frac{\partial \phi}{\partial y} + \frac{\partial A_y}{\partial t} & -c\frac{\partial A_x}{\partial y} - c\frac{\partial A_y}{\partial x} & -2c\frac{\partial A_y}{\partial y} & -c\frac{\partial A_z}{\partial y} - c\frac{\partial A_y}{\partial z} \\ -c\frac{\partial \phi}{\partial z} + \frac{\partial A_z}{\partial t} & -c\frac{\partial A_x}{\partial z} - c\frac{\partial A_z}{\partial x} & -c\frac{\partial A_y}{\partial z} - c\frac{\partial A_z}{\partial y} & -2c\frac{\partial A_z}{\partial z} \end{pmatrix}$$

3. $A^{\mu,\nu}$ written in terms of the classic gravitational, electric, and magnetic fields:

$$\begin{matrix} g_t & e_x - E_x & e_y - E_y & e_z - E_z \\ e_x + E_x & g_x & b_z - B_z & b_y + B_y \\ e_y + E_y & b_z + B_z & g_y & b_x - B_x \\ e_z + E_z & b_y - B_y & b_x + B_x & g_z \end{matrix}$$

2.5 Gauss' Law and Newton's Relativistic Gravitational Field

Method: $\frac{1}{2}$ (EM law + gravitational analog) + diagonal terms = field equations.

$$\rho_q - \rho_m = \frac{1}{2}(\vec{\nabla} \cdot \vec{E} + \vec{\nabla} \cdot \vec{e}) + \frac{\partial g_t}{c \partial t}$$
$$= \frac{1}{2}(- \frac{\partial^2 A_x}{\partial x \partial t} - c\frac{\partial^2 \phi}{\partial x^2} - \frac{\partial^2 A_y}{\partial y \partial t} - c\frac{\partial^2 \phi}{\partial y^2} - \frac{\partial^2 A_z}{\partial z \partial t} - c\frac{\partial^2 \phi}{\partial z^2}$$
$$+ \frac{\partial^2 A_x}{\partial x \partial t} - c\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 A_y}{\partial y \partial t} - c\frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 A_z}{\partial z \partial t} - c\frac{\partial^2 \phi}{\partial z^2} + \frac{\partial^2 \phi}{c \partial t^2}$$
$$= \frac{\partial^2 \phi}{c \partial t^2} - c\frac{\partial^2 \phi}{\partial x^2} - c\frac{\partial^2 \phi}{\partial y^2} - c\frac{\partial^2 \phi}{\partial z^2} = \Box^2 \phi$$

- Gauss' law results in the physical situation with no mass charge density and no divergence of the field $\vec{e}$.
- Newton's relativistic gravitational field equation results in the physical situation where there is no electric charge density and no divergence of the field $\vec{E}$.

Gauss' law indicates like electric charges repel, while Newton's field law implies like mass charges attract.

2.6 Ampere's Law with a Mass Current

Method: Same as previous.

$$\vec{J}_q - \vec{J}_m = \frac{1}{2}(- \frac{\partial \vec{E}}{c \partial t} + \frac{\partial \vec{e}}{c \partial t} + \vec{\nabla} \times \vec{B} + \nabla \boxtimes \vec{b}) + \vec{\nabla}_u g^u$$
$$= \frac{1}{2}(\frac{\partial^2 A_x}{c \partial t^2} + \frac{\partial^2 \phi}{\partial t \partial x}, \frac{\partial^2 A_y}{c \partial t^2} + \frac{\partial^2 \phi}{\partial t \partial y}, \frac{\partial^2 A_z}{c \partial t^2} + \frac{\partial^2 \phi}{\partial t \partial z})$$
$$+ \frac{1}{2}(\frac{\partial^2 A_x}{c \partial t^2} - \frac{\partial^2 \phi}{\partial t \partial x}, \frac{\partial^2 A_y}{c \partial t^2} - \frac{\partial^2 \phi}{\partial t \partial y}, \frac{\partial^2 A_z}{c \partial t^2} - \frac{\partial^2 \phi}{\partial t \partial z})$$
$$+ \frac{c}{2}(\frac{\partial^2 A_y}{\partial y \partial x} - \frac{\partial^2 A_x}{\partial y^2} + \frac{\partial^2 A_z}{\partial z \partial x} - \frac{\partial^2 A_x}{\partial z^2}, \frac{\partial^2 A_z}{\partial z \partial y} - \frac{\partial^2 A_y}{\partial z^2} + \frac{\partial^2 A_x}{\partial x \partial y} - \frac{\partial^2 A_y}{\partial x^2}, \frac{\partial^2 A_x}{\partial x \partial z} - \frac{\partial^2 A_z}{\partial x^2} + \frac{\partial^2 A_y}{\partial y \partial z} - \frac{\partial^2 A_z}{\partial y^2})$$
$$- \frac{c}{2}(\frac{\partial^2 A_y}{\partial y \partial x} + \frac{\partial^2 A_x}{\partial y^2} + \frac{\partial^2 A_z}{\partial z \partial x} + \frac{\partial^2 A_x}{\partial z^2}, \frac{\partial^2 A_z}{\partial z \partial y} + \frac{\partial^2 A_y}{\partial z^2} + \frac{\partial^2 A_x}{\partial x \partial y} + \frac{\partial^2 A_y}{\partial x^2}, \frac{\partial^2 A_x}{\partial x \partial z} + \frac{\partial^2 A_z}{\partial x^2} + \frac{\partial^2 A_y}{\partial y \partial z} + \frac{\partial^2 A_z}{\partial y^2})$$
$$- c(\frac{\partial^2 A_x}{\partial x^2}, \frac{\partial^2 A_y}{\partial y^2}, \frac{\partial^2 A_z}{\partial z^2})$$
$$= \Box^2 \vec{A}$$

- Ampere's law results in the physical situation where there is no mass current density, no gradient of the field g^u, and no boxed curl of b.
- A pure mass current equation results in the physical situation where there is no electric current density, no time change of the field $\vec{E}$, and no curl of the field $\vec{B}$.

2.7 Homogeneous Maxwell Equations

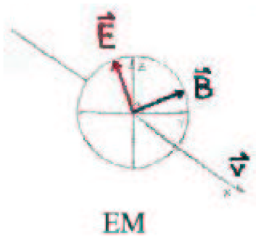
The homogeneous equations or vector identities are unchanged.

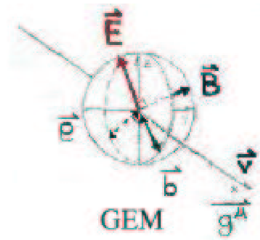
- $0 = \vec{\nabla} \cdot \vec{B} = \vec{\nabla} \cdot (c\vec{\nabla} \times \vec{A})$ No magnetic monopoles.
- $\vec{0} = \frac{\partial B}{c\partial t} + \vec{\nabla} \times \vec{E} = \frac{\partial}{\partial t}\vec{\nabla} \times \vec{A} - \vec{\nabla} \times \frac{\partial \vec{A}}{\partial t} - \vec{\nabla} \times c\vec{\nabla}\phi$ Faraday's law.

No obvious vector identity analogs for gravitational fields have been found yet.

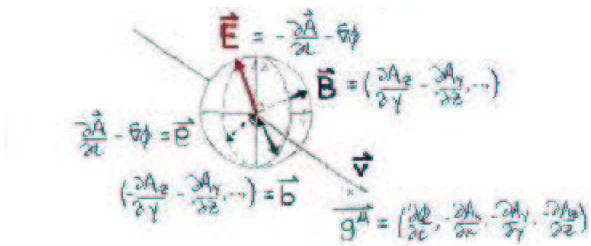
2.8 In Pictures

The electromagnetic fields $\vec{E}$ and $\vec{B}$ are **transverse** to the direction of motion. The gravitational field analogues $\vec{e}$ and $\vec{b}$ are **longitudinal**. There is a new 4-field g^{μ}:

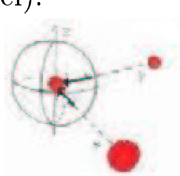


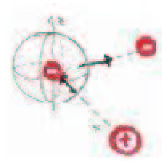


The 5 fields can be written in terms of the potentials:

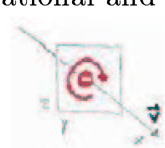


Newton's law of gravity (like charges attract) and Gauss' law (like charges repel):





Ampere's law for gravitational and electrical currents:



Chapter 3

Field Equation Solutions

3.1 4D-Wave Equation Solution in a Vacuum

1. Start with 4D-wave equation, no source:

   $\Box^2 A^\mu = 0$

2. Guess a solution with an inverse relativistic distance:

   $A^\mu = \frac{\sqrt{G}\,h}{c^2}\,((x^2+y^2+z^2-c^2t^2)^{-1}, 0, 0, 0) = \frac{\sqrt{G}\,h}{c^2}\,(\frac{1}{\sigma^2}, \vec{0})$

3. Take derivatives:

   $\frac{\partial}{c\partial t}(x^2+y^2+z^2-c^2t^2)^{-1} = +2ct\,(x^2+y^2+z^2-c^2t^2)^{-2}$

   $\frac{\partial}{\partial x}(x^2+y^2+z^2-c^2t^2)^{-1} = -2x\,(x^2+y^2+z^2-c^2t^2)^{-2}$

   $\frac{\partial}{\partial y}(x^2+y^2+z^2-c^2t^2)^{-1} = -2y\,(x^2+y^2+z^2-c^2t^2)^{-2}$

   $\frac{\partial}{\partial z}(x^2+y^2+z^2-c^2t^2)^{-1} = -2z\,(x^2+y^2+z^2-c^2t^2)^{-2}$

4. Take second derivatives:

   $\frac{\partial}{c\partial t}(+2t\sigma^{-4}) = +2(x^2+y^2+z^2-c^2t^2)^{-2} + 8c^2\,t^2(x^2+y^2+z^2-c^2t^2)^{-4}$

   $\frac{\partial}{\partial x}(-2x\sigma^{-4}) = -2(x^2+y^2+z^2-c^2t^2)^{-2} + 8x^2(x^2+y^2+z^2-c^2t^2)^{-4}$

   $\frac{\partial}{\partial y}(-2y\sigma^{-4}) = -2(x^2+y^2+z^2-c^2t^2)^{-2} + 8y^2(x^2+y^2+z^2-c^2t^2)^{-4}$

   $\frac{\partial}{\partial z}(-2z\sigma^{-4}) = -2(x^2+y^2+z^2-c^2t^2)^{-2} + 8z^2(x^2+y^2+z^2-c^2t^2)^{-4}$

5. Sum:

   $\frac{\partial^2 A_0}{c^2\partial t^2} - \frac{\partial^2 A_0}{\partial x^2} - \frac{\partial^2 A_0}{\partial y^2} - \frac{\partial^2 A_0}{\partial z^2} = 0 \quad$ QED

- $x^2+y^2+z^2-c^2t^2 = 0$ Practical value: Singularity is the lightcone.
- $\vec{\nabla}\frac{1}{x^2+y^2+z^2-c^2t^2} \neq f(\frac{1}{R^2})$ Practical problem: Derivative does not generate

an inverse square force law.

3.2 Normalization and Perturbations

Quantum mechanics cliché: normalize and look at perturbations for a weak field. Gravity is weak.

1. Normalization:
   - U(1)xsU(2)xsU(3) Unitary requirement of the standard model.
   - $\frac{A^\mu}{|A^\mu|} \rightsquigarrow -$ Dimensionless.
2. Perturbations:
   - $A \longrightarrow A' = A + k\delta$ Linear restoration.
   - k Spring constant (small number).
   - δ Variable.

3.3 Normalized, Perturbation Solution

1. Start with 4D-wave equation solution:
   $$A^\mu = \frac{\sqrt{G}\,h}{c^2}\,(\frac{1}{x^2+y^2+z^2-c^2\,t^2}, \vec{0}) = \frac{\sqrt{G}\,h}{c^2}\,(\frac{1}{\sigma^2}, \vec{0})$$
2. Normalize so that the magnitude of A^μ is one up to a constant:
   $$A^\mu = \frac{c}{\sqrt{G}}\,\frac{A^\mu}{|A^\mu|} = (\frac{c/\sqrt{G}}{\frac{x^2+y^2+z^2-c^2\,t^2}{\sigma^2}}, \vec{0}) = \frac{c}{\sqrt{G}}\,(1, \vec{0})$$
3. Perturb $x, y, z,$ and t linearly with a spring constant k:
   $$A^\mu = \frac{c}{\sqrt{G}}\,\frac{A'^\mu}{|A^\mu|} = (\frac{c/\sqrt{G}}{(\frac{1}{\sqrt{2}}+\frac{kx}{\sigma^2})^2+(\frac{1}{\sqrt{2}}+\frac{ky}{\sigma^2})^2+(\frac{1}{\sqrt{2}}+\frac{kz}{\sigma^2})^2-(\frac{1}{\sqrt{2}}+\frac{kct}{\sigma^2})^2}, \vec{0})$$
   $$= \frac{c}{\sqrt{G}}\,(\frac{\sigma^2}{\sigma'^2}, \vec{0}) \cong \frac{c}{\sqrt{G}}\,(1, \vec{0})$$

3.4 Derivative of the Normalized, Perturbation Solution

1. Start with the normalized, perturbation solution:

$$A^\mu = \frac{c}{\sqrt{G}}\frac{A'^\mu}{|A^\mu|} = \left(\frac{c/\sqrt{G}}{(\frac{1}{\sqrt{2}}+\frac{kx}{\sigma^2})^2+(\frac{1}{\sqrt{2}}+\frac{ky}{\sigma^2})^2+(\frac{1}{\sqrt{2}}+\frac{kz}{\sigma^2})^2-(\frac{1}{\sqrt{2}}+\frac{kct}{\sigma^2})^2}, \vec{0}\right) = \frac{c}{\sqrt{G}}\left(\frac{\sigma^2}{\sigma'^2}, \vec{0}\right)$$

2. Expand:

$$A^\mu = \left(\frac{c/\sqrt{G}}{(\frac{1}{2}+\frac{\sqrt{2}\,kx}{\sigma^2}+\frac{k^2x^2}{\sigma^4})+(\frac{1}{2}+\frac{\sqrt{2}\,ky}{\sigma^2}+\frac{k^2y^2}{\sigma^4})+(\frac{1}{2}+\frac{\sqrt{2}\,kz}{\sigma^2}+\frac{k^2z^2}{\sigma^4})-(\frac{1}{2}+\frac{\sqrt{2}\,kct}{\sigma^2}+\frac{k^2c^2t^2}{\sigma^4})}, \vec{0}\right)$$

$$= \frac{c}{\sqrt{G}}\left(\frac{\sigma^2}{\sigma'^2}, \vec{0}\right)$$

3. Take derivatives:

$$\frac{\partial\phi}{c\,\partial t} = \frac{c^2}{\sqrt{G}}\frac{\sigma^2}{\sigma'^4}k + O(k^2) \cong \frac{c^2}{\sqrt{G}}\frac{k}{\sigma^2} + O(k^2)$$

$$\frac{\partial\phi}{\partial R} = -\frac{c^2}{\sqrt{G}}\frac{\sigma^2}{\sigma'^4}k + O(k^2) \cong -\frac{c^2}{\sqrt{G}}\frac{k}{\sigma^2} + O(k^2)$$

- $\frac{1}{\sigma^2}$ An inverse square distance dependence.
- k A small number with units of distance.

3.5 Only Weak Gravity

A potential that only applies to gravity not EM will have derivatives only on the diagonal of the field strength tensor.

- The sign of the spring constant k does not effect the solution.
- The sign of the spring constant k does change the derivative of the potential to first order in k.
- A potential that only has derivatives along the diagonal can be constructed from two potentials that differ by spring constants that either constructively interfere to create a non-zero derivative, or destructively interfere to eliminate a derivative.

diagonal SHO $A^\mu = c/\sqrt{G}$

$$\left(\frac{1}{(\frac{1}{\sqrt{2}}+\frac{kx}{\sigma^2})^2+(\frac{1}{\sqrt{2}}+\frac{ky}{\sigma^2})^2+(\frac{1}{\sqrt{2}}+\frac{kz}{\sigma^2})^2-(\frac{1}{\sqrt{2}}+\frac{kct}{\sigma^2})^2} + \frac{1}{(\ldots-\frac{kx}{\sigma^2})^2+(\ldots-\frac{ky}{\sigma^2})^2+(\ldots-\frac{kz}{\sigma^2})^2-(\ldots+\frac{kct}{\sigma^2})^2},\right.$$

$$\frac{1}{(\frac{1}{\sqrt{2}}+\frac{kx}{\sigma^2})^2+(\frac{1}{\sqrt{2}}+\frac{ky}{\sigma^2})^2+(\frac{1}{\sqrt{2}}+\frac{kz}{\sigma^2})^2-(\frac{1}{\sqrt{2}}+\frac{kct}{\sigma^2})^2} + \frac{1}{(\ldots+\frac{kx}{\sigma^2})^2+(\ldots-\frac{ky}{\sigma^2})^2+(\ldots-\frac{kz}{\sigma^2})^2-(\ldots-\frac{kct}{\sigma^2})^2},$$

$$\frac{1}{(\frac{1}{\sqrt{2}}+\frac{kx}{\sigma^2})^2+(\frac{1}{\sqrt{2}}+\frac{ky}{\sigma^2})^2+(\frac{1}{\sqrt{2}}+\frac{kz}{\sigma^2})^2-(\frac{1}{\sqrt{2}}+\frac{kct}{\sigma^2})^2} + \frac{1}{(\ldots-\frac{kx}{\sigma^2})^2+(\ldots+\frac{ky}{\sigma^2})^2+(\ldots-\frac{kz}{\sigma^2})^2-(\ldots-\frac{kct}{\sigma^2})^2},$$

$$\left.\frac{1}{(\frac{1}{\sqrt{2}}+\frac{kx}{\sigma^2})^2+(\frac{1}{\sqrt{2}}+\frac{ky}{\sigma^2})^2+(\frac{1}{\sqrt{2}}+\frac{kz}{\sigma^2})^2-(\frac{1}{\sqrt{2}}+\frac{kct}{\sigma^2})^2} + \frac{1}{(\ldots-\frac{kx}{\sigma^2})^2+(\ldots-\frac{ky}{\sigma^2})^2+(\ldots+\frac{kz}{\sigma^2})^2-(\ldots-\frac{kct}{\sigma^2})^2}\right)$$

Notice the pattern for signs of k.

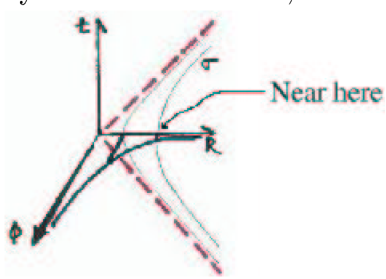
Take the contravariant derivative of this potential, keeping only the terms to first order in the spring constant k. Remember the contravariant derivative flips the sign of the 3-vector.

$$A^{\mu,\nu} \cong \frac{c^2}{\sqrt{G}} \frac{k}{\sigma^2} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

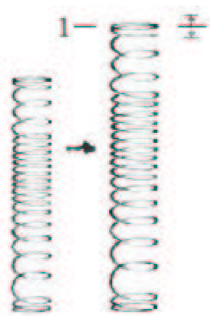
This is an identity matrix times $\frac{c^2}{\sqrt{G}} \frac{k}{\sigma^2}$, a simple result that required much work.

3.6 In Pictures

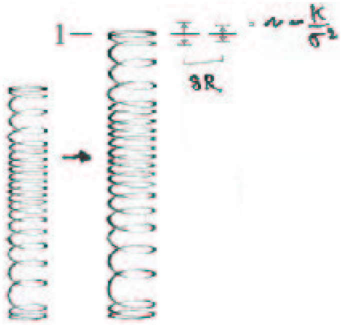
Study a solution near R, the relativistic distance σ:



Normalize the potential, look at linear perturbations.



Take the derivative of the normalized, linear perturbation solution:



Chapter 4
Relativistic Gravitational Force

4.1 EM Lorentz Force

The EM Lorentz force is caused by an electric charge moving in an antisymmetic EM field. The effect is to change momentum.

$F^{\mu}_{\mathrm{EM}} = q\,\frac{U_{\nu}}{c}(A^{\mu,\nu} - A^{\nu,\mu}) = \frac{\partial m\,U^{\mu}}{\partial\tau}$

- If the sign of electric charge is inverted ($q \longrightarrow -q$), F^{μ}_{EM} flips signs, so there are two distinguishable signs for electric charges.
- Like electrical charges repel due to the positive sign of the force.

4.2 EM to Gravity Analogy

- $-q \longrightarrow +\sqrt{G}\,m$ Electric charge to mass charge.
- Change field strength tensor's symmetry.
    1. $A - A \longrightarrow A + A$ Antisymmetric to symmetric tensor.
    2. $, \longrightarrow ;$ Derivatives to covariant derivatives.

4.3 Gravitational Lorentz Force

The gravitational Lorentz force is caused by a mass charge moving in a symmetric gravitational field. The effect is to change momentum.

$F^{\mu}_{G} = -\sqrt{G}\,m\,\frac{U_{\nu}}{c}(A^{\mu;\nu} + A^{\nu;\mu}) = \frac{\partial m\,U^{\mu}}{\partial\tau}$

- If the sign of inertial and mass charge is inverted ($m \longrightarrow -m$), F^{μ}_{G} is invariant, so there is one distinguishable sign for mass.
- Like mass charges attract due to the negative sign of the force.

4.4 Weak Field Force Law

1. Start from the gravitational Lorentz force law:

$$F_G^\mu = -\sqrt{G}\, m\, \frac{U_\nu}{c}(A^{\mu;\nu} + A^{\nu;\mu}) = \frac{\partial\, m\, U^\mu}{\partial \tau}$$

2. Assume local covariant coordinates (; $\rightarrow$,):
$$F_G^\mu = -\sqrt{G}\, m\, \frac{U_\nu}{c}(A^{\mu,\nu} + A^{\nu,\mu}) = \frac{\partial\, m\, U^\mu}{\partial \tau}$$

3. Recall the weak gravitational field strength tensor:
$$A^{\mu,\nu} \cong \frac{c^2}{\sqrt{G}}\, \frac{k}{\sigma 2} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

4. Substitute the normalized potential derivative (3) into the force law (2). Expand the velocity, $U_\nu \rightarrow (U_0, -\vec{U})$ and acceleration, $\frac{\partial\, U^\mu}{\partial \tau} \rightarrow (\frac{\partial\, U_0}{\partial \tau}, \frac{\partial\, \vec{U}}{\partial \tau})$:
$$F_G^\mu = -\, m\, c^2\, (\frac{U_0}{c}, -\frac{\vec{U}}{c}) \begin{pmatrix} \frac{k}{\sigma^2} & 0 \\ 0 & \frac{k}{\sigma^2} \end{pmatrix} = (\frac{\partial m U_0}{\partial \tau}, \frac{\partial\, m\, \vec{U}}{\partial \tau})$$

5. Contract the velocity with the derivative of the potential:
$$F_G^\mu = m(-\frac{c\, k}{\sigma^2}\, U_0, \frac{c\, k}{\sigma^2}\, \vec{U}) = (\frac{\partial m U_0}{\partial \tau}, \frac{\partial\, m\, \vec{U}}{\partial \tau})$$

6. Substitute $c^2 \tau^2$ for $-\sigma^2$:
$$F_G^\mu = m(\frac{k}{\tau^2}\, \frac{U_0}{c}, -\frac{k}{\tau^2}\, \frac{\vec{U}}{c}) = (\frac{\partial m U_0}{\partial \tau}, \frac{\partial\, m\, \vec{U}}{\partial \tau})$$

A first-order differential equation!

4.5 Exact Weak Field Force Solution

The gravitational Lorentz force for a weak field is a first order differential equation that can be solved exactly.

1. Start from the weak field force law:
$$F_G^\mu = m(\frac{k}{\tau^2}\, \frac{U_0}{c}, -\frac{k}{\tau^2}\, \frac{\vec{U}}{c}) = (\frac{\partial m U_0}{\partial \tau}, \frac{\partial\, m\, \vec{U}}{\partial \tau})$$

2. Apply the chain rule to the cause terms. Assume $U_0\, \frac{\partial\, m}{\partial \tau} = \vec{U}\, \frac{\partial\, m}{\partial \tau} = 0$.
   Collect terms on one side:
$$(m\, \frac{\partial U_0}{\partial \tau} - m \frac{k}{\tau^2}\, \frac{U_0}{c}, m\, \frac{\partial\, \vec{U}}{\partial \tau} + m\, \frac{k}{\tau^2}\, \frac{\vec{U}}{c}) = 0$$

3. Assume the equivalence principle. Drop m:
$$(\frac{\partial U_0}{\partial \tau} - \frac{k}{\tau^2}\, \frac{U_0}{c}, \frac{\partial\, \vec{U}}{\partial \tau} + \frac{k}{\tau^2}\, \frac{\vec{U}}{c}) = 0$$

4. Solve for velocity:
$$(U_0, \vec{U}) = (c_0\, e^{-\frac{k}{c\tau}}, \vec{C}_{1-3}\, e^{+\frac{k}{c\tau}})$$

5. Contract the velocity solution:
$$U^\mu\, U_\mu = c_0^2\, e^{-2\frac{k}{c\tau}} - \vec{C}_{1-3}\, e^{+2\frac{k}{c\tau}}$$

6. For flat spacetime ($k \rightarrow 0$, or $\tau \rightarrow \infty$), there are four constraints on the contracted velocity solution:

$$U^{\mu}U_{\mu} = (c\,\frac{\partial t}{\partial \tau}, \frac{\partial \vec{R}}{\partial \tau})(c\,\frac{\partial t}{\partial \tau}, -\frac{\partial \vec{R}}{\partial \tau}) = \frac{c^2(\partial t)^2 - (\partial R)^2}{(\partial t)^2 - (\frac{\partial R}{c})^2} = c^2$$

$$\text{True if and only if:} \quad c_0 = c\,\frac{\partial t}{\partial \tau} = U_{0\,\mathrm{flat}}, \quad \vec{C}_{1-3} = \frac{\partial \vec{R}}{\partial \tau} = \vec{U}_{\mathrm{flat}}$$

7. Substitute $c\,\frac{\partial t}{\partial \tau}$ for c_0, $\frac{\partial \vec{R}}{\partial \tau}$ for $\vec{C}_{1-3}$ into the contracted velocity solution. Multiply through by $(\frac{\partial \tau}{c})^2$:

$$(\partial \tau)^2 = e^{-2\frac{k}{c\tau}}(\partial t)^2 - e^{+2\frac{k}{c\tau}}(\frac{\partial \vec{R}}{c})^2$$

- $k = 0$, or $\tau \to \infty$ — Flat spacetime.
- $e^{-2\frac{k}{c\tau}} \neq 1$ — Curved spacetime.

4.6 Exact Weak Field Force Solution Applied

Apply to a weak, spherically symmetric, gravitational system.

- $k = \frac{GM}{c^2}$ — Spring constant k is a geometric mass.
- $\sigma^2 = R^2 - (ct)^2 = R'^2$ — Static field approximated by R'.
- $|\sigma| = |c\tau| = R$ — σ and $c\tau$ have the same magnitude.
- $(+i\sigma)^2 = (+c\tau)^2$ — To make a real metric, choose σ to be imaginary.

Plug into the exact solution:

$$(\partial \tau)^2 = e^{-2\frac{GM}{c^2 R}}(\partial t)^2 - e^{+2\frac{GM}{c^2 R}}(\frac{\partial \vec{R}}{c})^2$$

4.7 Compare Metrics: Schwarzschild to GEM

The Schwarzschild metric is a solution of general relativity for a neutral, non-rotating, spherically symmetric source mass. Write out the Taylor series expansion of the Schwarzschild and GEM metrics in isotropic coordinates to third order in $\frac{GM}{c^2 R}$.

1. Schwarzschild metric:

$$(\partial \tau)^2 = (\underline{1 - 2\frac{GM}{c^2 R} + 2(\frac{GM}{c^2 R})^2} - \frac{3}{2}(\frac{GM}{c^2 R})^3)(\partial t)^2$$
$$- (\underline{1 + 2\frac{GM}{c^2 R}} + \frac{3}{2}(\frac{GM}{c^2 R})^2 + \frac{1}{2}(\frac{GM}{c^2 R})^3)(\partial \vec{R})^2$$

2. GEM metric:

$$(\partial \tau)^2 = (\underline{1 - 2\frac{GM}{c^2 R} + 2(\frac{GM}{c^2 R})^2} - \frac{4}{3}(\frac{GM}{c^2 R})^3)(\partial t)^2$$
$$- (\underline{1 + 2\frac{GM}{c^2 R}} + 2(\frac{GM}{c^2 R})^2 + \frac{4}{3}(\frac{GM}{c^2 R})^3)(\frac{\partial \vec{R}}{c})^2$$

The ten parameterized post-Newtonian (PPN) parameters are identical: $\gamma = 1$, $\beta = 1$, $\xi = \alpha_{1-3} = \zeta_{1-4} = 0$.

The five underlined terms have been confirmed experimentally in weak field tests, including:

- Light bending around the Sun.
- Perihelion shift of Mercury.
- Time delay in radar reflections off of planets.

Strong field tests, such as the lost of energy by gravity waves in binary pulsars, depend on order 2.5 PPN parameters, which are similar, but not identical.

4.8 The Equivalence Principle

There are three types of mass in a gravitational force equation: inertial mass (m_i), passive gravitational mass (m_p), and active gravitational mass (M_a).

- Eötvös experiments show inertial and passive gravitational masses are equal ($m_i = m_p$).
- Kreuzer's experiment and measurements of the moon indicate passive and active gravitational masses are equal ($m_p = M_a$).

Because the ten PPN parameters are identical, both the Schwarzschild and GEM metric are consistent with these results.

4.9 Dogma Dogfight

General relativity dogma:

- Field equations are rank 2.
- Field equations are nonlinear.
- Gravity binds to energy-momentum.
- The Equivalence Principle:
  $m_p + \sum \eta^a E^a/c^2 = m_i + \sum \eta^a E^a/c^2$

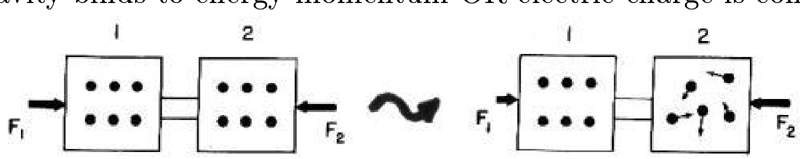
Unified field dogma:

- Field equations are rank 1.
- Field equations are linear.
- Gravity binds to rest mass.
- The Equivalence Principle:
  $m_p = m_i$

No data directly demonstrates the nonlinearity of gravity (too weak).

4.10 Thought Experiment Dilemma

Gravity binds to energy-momentum OR electric charge is conserved (not both).



All the rest mass of one particle becomes the kinetic energy of five others.

If gravity is a function of rest mass only, then the system accelerates without any external forces applied. Therefore gravity is a function of energy-momentum.

Make one specification: every particle is an electron, with 511 keV of rest mass and -1 unit of electric charge.

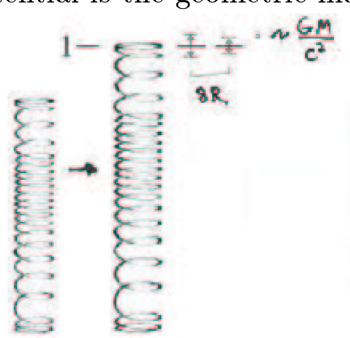
Box 2 has 511 keV of kinetic energy. Electric charge cannot be split into fifths. The box with 6 charges now has 5 moving charges. One unit of electric charge has been destroyed.

The data for electric charge conservation is exceptionally good.

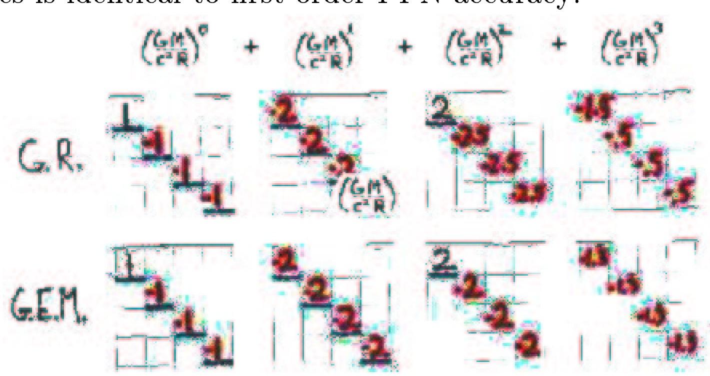
The thought experiment is flawed, so gravity does not bind to energy-momentum.

4.11 In Pictures

The spring constant k of the derivative of the normalized, linear perturbation potential is the geometric mass:



The Taylor series expansion for the Schwarzschild metric in isotropic coordinates is identical to first order PPN accuracy:



Chapter 5
Classical Gravitational Force

5.1 Breaking Spacetime Symmetry

Spacetime symmetry must be broken to go from the relativistic weak gravitational Lorentz 4-force to a classical 3-force for both cause and effect.

Contrast the relativistic geometry of Minkowski spacetime with the classic geometry of Newtonian absolute space and time.

Geometry	**Minkowski Spacetime**	**Newtonian Space and Time**
Utility	True, Elegant	Accurate, Practical
Interval	$(\partial\tau)^2 = (dt)^2 - (dR/c)^2$	$\text{distance}^2 = dR^2 \neq f(t)$
Velocity	$(U_0, \vec{U}) = (c\,\frac{\partial t}{\partial\tau}, \frac{\partial\vec{R}}{\partial\tau})$	$(\mathbb{U}_0, \vec{\mathbb{U}}) \equiv (\frac{\partial t}{\partial\lvert R\rvert}, c\frac{\partial\vec{R}}{\partial\lvert R\rvert}) = (0, c\,\hat{R})$
Acceleration	$(\frac{\partial U_0}{\partial\tau}, \frac{\partial\vec{U}}{\partial\tau}) = (c\,\frac{\partial^2 t}{\partial\tau^2}, \frac{\partial^2\vec{R}}{\partial\tau^2})$	$(\frac{\partial\mathbb{U}_0}{\partial\lvert R\rvert}, \frac{\partial\vec{\mathbb{U}}}{\partial\lvert R\rvert}) = (0, c^2\,\frac{\partial^2\vec{R}}{\partial\lvert R\rvert^2})$

5.2 Derive Newton's Gravitational Law

1. Start from the weak field gravitational Lorentz 4-force:
   $F_G^\mu = m\,(\frac{k}{\tau^2}\,\frac{U_0}{c}, -\,\frac{k}{\tau^2}\,\frac{\vec{U}}{c}) = (\frac{\partial\,m U_0}{\partial\tau}, \frac{\partial\,m\vec{U}}{\partial\tau})$
2. Apply the chain rule to the cause terms.
   Assume inertial mass is constant in spacetime, $U_0\,\frac{\partial m}{\partial\tau} = \vec{U}\,\frac{\partial m}{\partial\tau} = 0$:
   $F_G^\mu = m\,(\frac{k}{\tau^2}\,\frac{U_0}{c}, -\,\frac{k}{\tau^2}\,\frac{\vec{U}}{c}) = (m\,\frac{\partial U_0}{\partial\tau}, m\,\frac{\partial\vec{U}}{\partial\tau})$
3. Break spacetime symmetry:
   - $(U_0, \vec{U}) \longrightarrow (\mathbb{U}_0, \vec{\mathbb{U}}) = (0, c\,\hat{R})$
   - $(\frac{\partial U_0}{\partial\tau}, \frac{\partial\vec{U}}{\partial\tau}) \longrightarrow (0, c^2\,\frac{\partial^2\vec{R}}{\partial\lvert R\rvert^2})$

   $F_G^\mu = m\,(0, -\,\frac{k}{\tau^2}\,\hat{R}) = (0, m c^2\,\frac{\partial^2\vec{R}}{\partial\lvert R\rvert^2})$
4. Assume a gravitational spring constant $(k = \frac{GM}{c^2})$:
   $F_G^\mu = (0, -\,\frac{GMm}{c^2\tau^2}\,\hat{R}) = (0, m c^2\,\frac{\partial^2\vec{R}}{\partial\lvert R\rvert^2})$
5. Substitute: σ^2 for $-\,c^2\,\tau^2$ in the cause term.
   Substitute: $-\,c^2\,(\frac{\partial}{\partial\tau})^2$ for $(\frac{\partial}{\partial\lvert R\rvert})^2 = (\frac{\partial}{\partial\sigma})^2$ in the effect term.
   $F_G^\mu = (0, \frac{GMm}{\sigma^2}\,\hat{R}) = (0, -\,m\,\frac{\partial^2\vec{R}}{\partial\tau^2})$
6. Assume the static field approximation: $\sigma^2 = R^2 - (ct)^2 \cong R'^2$.

Assume the low-speed approximation: $\frac{\partial^2}{\partial\tau^2} \cong \frac{\partial^2}{\partial t^2}$:
$F_G^\mu = (0, \frac{GMm}{R^2}\hat{R}) = (0, -m\frac{\partial^2 \vec{R}}{\partial t^2})$ QED

5.3 New Stable Constant-Velocity Solution

1. Start from the weak field gravitational Lorentz 4-force:
   $F_G^\mu = m(\frac{k}{\tau^2}\frac{U_0}{c}, -\frac{k}{\tau^2}\frac{\vec{U}}{c}) = (\frac{\partial mU_0}{\partial\tau}, \frac{\partial m\vec{U}}{\partial\tau})$
2. Apply the chain rule to the cause terms.
   Assume $m\frac{\partial U_0}{\partial\tau} = m\frac{\partial\vec{U}}{\partial\tau} = 0$ (assume velocity is constant):
   $F_G^\mu = m(\frac{k}{\tau^2}\frac{U_0}{c}, -\frac{k}{\tau^2}\frac{\vec{U}}{c}) = (U_0\frac{\partial m}{\partial\tau}, \vec{U}\frac{\partial m}{\partial\tau})$
3. Break spacetime symmetry: $(U_0, \vec{U}) \longrightarrow (\mathbb{U}_0, \vec{\mathbb{U}}) = (0, c\hat{R})$.
   $F_G^\mu = m(0, -\frac{k}{\tau^2}\hat{R}) = (0, \frac{\partial m}{\partial\tau}c\hat{R})$
4. Assume a gravitational spring constant ($k = \frac{GM}{c^2}$):
   $F_G^\mu = (0, -\frac{GMm}{c^2\tau^2}\hat{R}) = (0, \frac{\partial m}{\partial\tau}c\hat{R})$
5. Collect terms on one side:
   $(c\frac{\partial m}{\partial\tau} + \frac{GMm}{c^2\tau^2})(0, \hat{R}) = 0$
6. Solve for m:
   $m = m_o e^{\frac{GM}{c^3\tau}}$
7. Assume a static field, $|\sigma| = |c\tau|$=R, and sigma is imaginary. Substitute:
   $m = m_0 e^{\frac{GM}{c^2 R}}$

Gravity causes a change in momentum, the product of inertial mass and velocity. If the velocity is constant, then the change is in the distribution of inertial mass in space.

5.4 Rotation Profile of Thin Spiral Galaxies

- Flat velocity profile
  After attaining a maximal speed consistent with Newton's law of gravity near the core, the velocity profile stays flat with increasing distance. Newton's law predicts a "Keplerian" decline for the velocity of the outer stars.
- Stability.
  Thin spiral galaxies are mathematically unstable to small disturbances along the axis of rotation which should lead to their collapse.
- Mass distribution.
  The mass distribution in space falls off exponential with distance from the center of the galaxy.

The rotation profile for thin spiral galaxies uses Newton's gravitational law near the core. For the larger radii where the velocity is constant but the mass falls off exponentially, the new stable constant-velocity solution may apply. At this time I have not been able to numerically check this proposal.

5.5 Early Big Bang

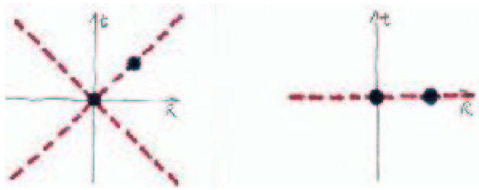
Big bang cosmology has two big problems:

- The horizon problem.
  All $\sim 10^{83}$ separate, independent spacetime volumes of the early Universe must travel at the same velocity to create the uniform black body radiation spectrum seen in the cosmic background radiation.
- The flatness problem.
  The initial conditions must be tuned to one part in $\sim 10^{55}$ so the mathematically unstable solution lasts 10^{10} years.

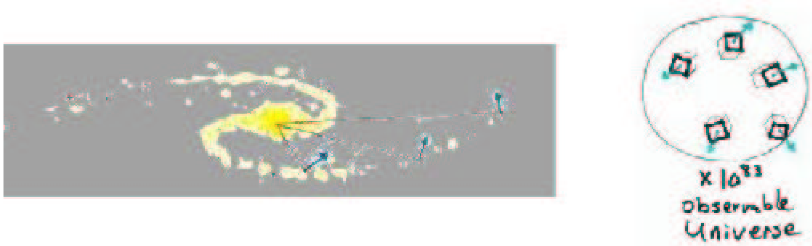
At this time, I do not know how people think the mass distribution changed in space during the earliest phases of the big bang. If the distribution could be described with an exponential function, then the cause of early expansion could be the new classical constant-velocity solution for gravity.

5.6 In Pictures

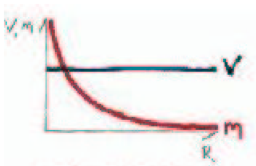
Minkowski spacetime versus Newtonian space and time



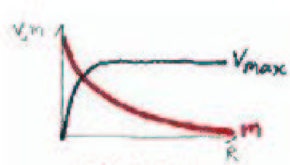
Newtonian gravity cannot explain the rotation profile of thin galaxies or the early big bang.



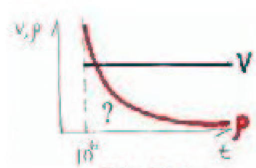
The new stable constant-velocity solution has the same momentum profile (velocity * mass) as the spiral galaxy and [perhaps] the early big bang.



New Solution



Spiral Galaxy



Big Bang

Chapter 6
Quantization

6.1 Momentum from the EM Lagrange Density

1. Start with the EM Lagrange density written without indices.
$$\mathfrak{L}_{\mathrm{EM}} = -\frac{\rho_m}{\gamma} - \frac{1}{c}J_q^{\mu}A_{\mu} - \frac{1}{4c^2}(A^{\mu.\nu} - A^{\nu,\mu})(A_{\mu,\nu} - A_{\nu,\mu})$$
$$= -\rho_m(\sqrt{1 - (\frac{\partial x}{c\partial t})^2 - (\frac{\partial y}{c\partial t})^2 - (\frac{\partial z}{c\partial t})^2}$$
$$-\frac{1}{c}\rho_q(c\phi - \frac{\partial x}{\partial t}A_x - \frac{\partial x}{\partial t}A_y - \frac{\partial z}{\partial t}A_z)$$
$$-\frac{1}{2}((\quad -(\frac{\partial \phi}{\partial x})^2 - (\frac{\partial \phi}{\partial y})^2 - (\frac{\partial \phi}{\partial z})^2 - (\frac{\partial A_x}{c\partial t})^2 \quad + (\frac{\partial A_x}{\partial y})^2 + (\frac{\partial A_x}{\partial z})^2$$
$$-(\frac{\partial A_y}{c\partial t})^2 + (\frac{\partial A_y}{\partial x})^2 \quad + (\frac{\partial A_y}{\partial z})^2 - (\frac{\partial A_z}{c\partial t})^2 + (\frac{\partial A_z}{\partial x})^2 + (\frac{\partial A_z}{\partial y})^2$$
$$-2\frac{\partial A_x}{c\partial t}\frac{\partial \phi}{\partial x} - 2\frac{\partial A_y}{c\partial t}\frac{\partial \phi}{\partial y} - 2\frac{\partial A_z}{c\partial t}\frac{\partial \phi}{\partial z} - 2\frac{\partial A_y}{\partial z}\frac{\partial A_z}{\partial y} - 2\frac{\partial A_z}{\partial x}\frac{\partial A_x}{\partial z} - 2\frac{\partial A_x}{\partial y}\frac{\partial A_y}{\partial x})$$

2. Calculate momentum:
$$\pi^{\mu} = h\sqrt{G}\frac{\partial \mathfrak{L}}{\partial \frac{\partial A^{\mu}}{c\partial t}} = h\sqrt{G}\,(0, \frac{\partial A_x}{c\partial t} + \frac{\partial \phi}{\partial x}, \frac{\partial A_y}{c\partial t} + \frac{\partial \phi}{\partial y}, \frac{\partial A_z}{c\partial t} + \frac{\partial \phi}{\partial z})$$

3. Momentum cannot be made into an operator:
$[A_t, \pi_t]|\psi\rangle = [A_t, 0]|\psi\rangle = 0$ Energy commutes with its conjugate.

6.2 Quantize EM Fields by Fixing the Gauge

An EM gauge is a relationship between ϕ and $\vec{A}$ that does not change the Maxwell equations. Examples:

- $\vec{\nabla} \cdot \vec{A} = 0$ or $\mathrm{trace}(A^{\mu,\nu}) = \frac{\partial \phi}{\partial t}$ Coulomb gauge.
- $\frac{\partial \phi}{c\partial t} + \vec{\nabla} \cdot \vec{A} = 0$ or $\mathrm{trace}(A^{\mu,\nu}) = 0$ Lorenz gauge.

For EM ignoring gravity, one is free to assign arbitrary values to the diagonal of the field strength tensor.

Fix the Lorenz gauge in the EM Lagrange density.

1. Start with the Gupta-Bleuler Lagrange density written without indices:

$$\mathfrak{L}_{G-B} = -\frac{\rho_m}{\gamma} - \frac{1}{c} J_q^{\mu} A_{\mu} - \frac{1}{2c^2}(A^{\mu}{}_{,\mu})^2 - \frac{1}{4c^2}(A^{\mu.\nu} - A^{\nu,\mu})(A_{\mu,\nu} - A_{\nu,\mu})$$
$$= -\rho_m(\sqrt{1 - (\frac{\partial x}{c\partial t})^2 - (\frac{\partial y}{c\partial t})^2 - (\frac{\partial z}{c\partial t})^2}$$
$$-\frac{1}{c}\rho_q(c\phi - \frac{\partial x}{\partial t}A_x - \frac{\partial x}{\partial t}A_y - \frac{\partial z}{\partial t}A_z)$$
$$-\frac{1}{2}((\frac{\partial \phi}{c\partial t})^2 - (\frac{\partial \phi}{\partial x})^2 - (\frac{\partial \phi}{\partial y})^2 - (\frac{\partial \phi}{\partial z})^2 - (\frac{\partial A_x}{c\partial t})^2 + (\frac{\partial A_x}{\partial x})^2 + (\frac{\partial A_x}{\partial y})^2 + (\frac{\partial A_x}{\partial z})^2$$
$$-(\frac{\partial A_y}{c\partial t})^2 + (\frac{\partial A_y}{\partial x})^2 + (\frac{\partial A_y}{\partial y})^2 + (\frac{\partial A_y}{\partial z})^2 - (\frac{\partial A_z}{c\partial t})^2 + (\frac{\partial A_z}{\partial x})^2 + (\frac{\partial A_z}{\partial y})^2 + (\frac{\partial A_z}{\partial z})^2$$
$$-2\frac{\partial A_x}{c\partial t}\frac{\partial \phi}{\partial x} - 2\frac{\partial A_y}{c\partial t}\frac{\partial \phi}{\partial y} - 2\frac{\partial A_z}{c\partial t}\frac{\partial \phi}{\partial z} - 2\frac{\partial A_y}{\partial z}\frac{\partial A_z}{\partial y} - 2\frac{\partial A_z}{\partial x}\frac{\partial A_x}{\partial z} - 2\frac{\partial A_x}{\partial y}\frac{\partial A_y}{\partial x})$$
$$+2\frac{\partial \phi}{c\partial t}\frac{\partial A_x}{\partial x} + 2\frac{\partial \phi}{c\partial t}\frac{\partial A_y}{\partial y} + 2\frac{\partial \phi}{c\partial t}\frac{\partial A_z}{\partial z} + 2\frac{\partial A_x}{\partial x}\frac{\partial A_y}{\partial y} + 2\frac{\partial A_x}{\partial x}\frac{\partial A_z}{\partial z} + 2\frac{\partial A_y}{\partial y}\frac{\partial A_z}{\partial z})$$

2. Calculate momentum:
$$\pi^{\mu} = h\sqrt{G}\frac{\partial \mathfrak{L}}{\partial \frac{\partial A^{\mu}}{c\partial t}} = h\sqrt{G}\,(-\frac{\partial \phi}{c\partial t} - \vec{\nabla}\cdot\vec{A}, \frac{\partial A_x}{c\partial t} + \frac{\partial \phi}{\partial x}, \frac{\partial A_y}{c\partial t} + \frac{\partial \phi}{\partial y}, \frac{\partial A_z}{c\partial t} + \frac{\partial \phi}{\partial z})$$
3. Momentum can be made into an operator.

Using the Euler-Lagrange equation, the equations of motion are identical to those of $\mathfrak{L}_{\text{GEM}}$, except there is only one current.

$$J_q^{\mu} = (\frac{\partial^2}{c\,\partial t^2} - c\nabla^2)A^{\mu}$$

6.3 Gupta-Bleuler Quantization Method

Results of the Gupta-Bleuler quantization method:

- Four modes of transmission:
  1. Two transverse waves.
  2. One longitudinal wave.
  3. One scalar wave.
- Transverse waves are photons for EM.
- Scalar mode of transmission is called a "scalar photon".
- "Supplementary condition" is imposed to eliminate scalar and longitudinal photons as real particles, so they are always virtual.

6.4 Skeptical Analysis of Fixing the Lorenz Gauge

1. A scalar photon is an oxymoron. Photons must transform like vectors, even if photons happen to be virtual.
2. Eliminating an oxymoron cannot justify the supplementary condition.
3. A better interpretation for the 4D-wave equations of motion may exist.

6.5 Momentum from GEM Lagrange Density

1. Start with the GEM Lagrange density written without indices:

$$\begin{aligned}\mathfrak{L}_{\mathrm{GEM}} = &- \rho_m \left(\sqrt{1 - \left(\frac{\partial x}{c\partial t}\right)^2 - \left(\frac{\partial y}{c\partial t}\right)^2 - \left(\frac{\partial z}{c\partial t}\right)^2}\right. \\ &- \frac{1}{c}(\rho_q - \rho_m)\left(c\phi - \frac{\partial x}{\partial t}A_x - \frac{\partial x}{\partial t}A_y - \frac{\partial z}{\partial t}A_z\right) \\ &- \frac{1}{2}\left(\left(\frac{\partial \phi}{c\partial t}\right)^2 - \left(\frac{\partial \phi}{\partial x}\right)^2 - \left(\frac{\partial \phi}{\partial y}\right)^2 - \left(\frac{\partial \phi}{\partial z}\right)^2 - \left(\frac{\partial A_x}{c\partial t}\right)^2 + \left(\frac{\partial A_x}{\partial x}\right)^2 + \left(\frac{\partial A_x}{\partial y}\right)^2 + \left(\frac{\partial A_x}{\partial z}\right)^2\right. \\ &- \left(\frac{\partial A_y}{c\partial t}\right)^2 + \left(\frac{\partial A_y}{\partial x}\right)^2 + \left(\frac{\partial A_y}{\partial y}\right)^2 + \left(\frac{\partial A_y}{\partial z}\right)^2 - \left(\frac{\partial A_z}{c\partial t}\right)^2 + \left(\frac{\partial A_z}{\partial x}\right)^2 + \left(\frac{\partial A_z}{\partial y}\right)^2 + \left(\frac{\partial A_z}{\partial z}\right)^2\end{aligned}$$

2. Calculate momentum:

$$\pi^{\mu} = h\sqrt{G}\frac{\partial \mathfrak{L}}{\partial \frac{\partial A^{\mu}}{c\partial t}} = h\sqrt{G}\left(-\frac{\partial \phi}{c\partial t}, \frac{\partial A_x}{c\partial t}, \frac{\partial A_y}{c\partial t}, \frac{\partial A_z}{c\partial t}\right)$$

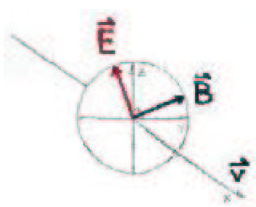
3. Momentum can be made into an operator.

6.6 GEM Quantization

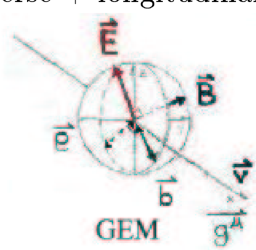
- There are at least two spin fields, one even, one odd.
- Four modes of transmission:
    1. Two transverse waves.
    2. One longitudinal wave.
    3. One scalar wave.
- Transverse waves are photons for EM, like charges repel.
- Longitudinal and scalar modes are gravitons with even spin because like charges attract.
- GEM predicts the polarization will not be transverse, in contrast to general relativity. Gravitational wave experiments (LIGO, etc) may be able to confirm one theory or the other.

6.7 In Pictures

Quantize transverse modes versus a transverse + longitudinal + scalar modes.



EM



GEM

Chapter 7
The Standard Model

7.1 Group Theory

Way to organize symmetry systematically.

Definition: A set S with a binary operation ($\times$ or $+$) such that $s_1 \times s_2 \in S$ for all possible pairs of elements in S. If $s_1 \times s_2 = s_2 \times s_1$, the group is Abelian, otherwise it is non-Abelian. A group has:

- An identity.
- An inverse for every element.
- Associative law holds.

7.2 The Standard Model

Predicts patterns of all subatomic particles and three of four forces in Nature:

- U(1) EM.
- SU(2) Weak force.
- SU(3) Strong force.

Says nothing about mass or gravity.

7.3 The Standard Model Lagrange Density

Describes interactions of three forces in a volume.

$\mathfrak{L}_{\mathrm{SM}} = \bar{\psi}\gamma^\mu D_\mu \psi$

$D_\mu = \partial_\mu - i\,g_{\mathrm{EM}} Y A_\mu - i\,g_{\mathrm{weak}} \frac{\tau^a}{2} W_\mu^a - i\,g_{\mathrm{strong}} \frac{\lambda^b}{2} G_\mu^b$

- γ^μ Spinor matrix.
- $g_{\ldots}$ Coupling constant to force.
- Y Generator of U(1) symmetry.
- $\tau^{a(1-3)}$ Generator of SU(2) symmetry.
- $\lambda^{b(1-8)}$ Generator of SU(3) symmetry.
- A_μ, W_μ^a, G_μ^b Complex-valued 4-potentials, two with internal symmetries.

7.4 Defining the Multiplication Operator

Four components:

1. $(a, bi)^* = (a, -bi)$ Complex conjugation.
2. $(\phi, \vec{A})^p = (\phi, -\vec{A})$ Parity operator.
3. $g_{\mu\nu}$ Metric tensor.
4. $\frac{A^\mu}{|A|}$ Potentials normalized to themselves.

Define multiplication of 4-potentials in the standard model as:

$$\frac{A^\mu}{|A|}\frac{A^{\nu*p}}{|A|}g_{\mu\nu} = \frac{g_{\mathrm{tt}}|A_t|^2 - g_{\mathrm{xx}}|A_x|^2 - g_{\mathrm{yy}}|A_y|^2 - g_{\mathrm{zz}}|A_z|^2 - g_{\mu\nu}|A^\mu A^\nu|_{\mu\neq\nu}}{|A|^2}$$

7.5 Multiplication Operator in Spacetime

- $\frac{A^\mu}{|A|}\frac{A^{\nu*p}}{|A|}g_{\mu\nu} = 1.0$ In flat spacetime.
- $\frac{A^\mu}{|A|}\frac{A^{\nu*p}}{|A|}g_{\mu\nu} = 1.0 + \delta$ In curved spacetime.

In curved spacetime, mass breaks U(1), SU(2), and SU(3) symmetry in a precise way (circles get larger).

Y, τ^a, λ^b and the Higgs particle are not needed.

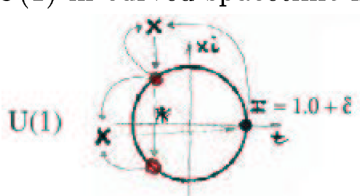
7.6 Interacting Particles for Gravity

No new symmetry was added to the standard model, so no new particle can be added. Instead, every particle can "act like an even-spin graviton" when it is involved with a distance measurement of the field.

Photons are mass charge neutral, like gravitons. No exchange of virtual gravitons is needed to follow geodesics in curved spacetime for photons or gravitons. This is a different way to say the gravitational field is linear.

7.7 In Pictures

U(1) in curved spacetime has an absolute value greater than 1.0.



SU(2) in curved spacetime has a norm greater than 1.0.