Unifying Gravity and EM by Generalizing EM

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Abstract

Surely I must be joking, since this must have been tried and shown to fail! Alas, the literature is sparse on the topic. One published fear is that it is not possible to construct a 4-vector field equation where like charges attract. That objection is easily overcome by one well-placed minus sign in the Lagrange density. A second known but not often discussed problem is to find a potential that is physically relevant. It is trivial to find a relativistic 1/distance^2 potential whose derivative when used in a force equation is non-physical. Using 4D normalized linear perturbation theory near a classical 1/R potential results in a force with the correct distance dependence.

A dynamic metric equation (the Rosen or exponential metric) solves the rank 1 field equations. This metric is consistent with the weak and strong equivalence principle, as well as the classical tests of general relativity to first order parameterized post-Newtonian accuracy. It should be distinguishable from the Schwarzschild solution at second order PPN accuracy, making this proposal possible to test. A key technical question is whether the proposal is "background free", which may make for interesting discussions.

It may even be possible to quantize the 4D field equations using 2 spin fields: spin 1 for EM and spin 2 for gravity. A source-free linear theory consistent with the equivalence principle is a good thing!

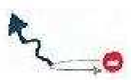
Table of contents

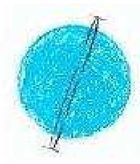
The Big Picture: A 4D Slinky

Light and gravity behave like a simple harmonic oscillator, or slinky, in four dimensions.

- Light is created by electrons wobbling (transverse modes).
- The Earth has wobbled around the Sun 4 billion times (scalar and longitudinal modes).

A thought experiment: Imagine a cup that could hold neutrinos still. Turn that cup over. The neutrinos would wobble through the Earth as a SHO, cycling to the other side of the Earth and back every 88 minutes. Because the acceleration is in the direction of the velocity, this is a longitudinal wave.





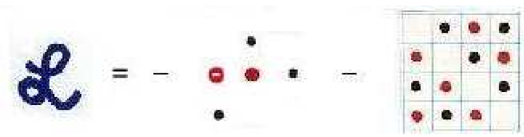
Field Equations and the Exponential Metric

EM Lagrange Density

The Maxwell equations are the most successful field equations in physics.

- Like electric charges **repel**.
- **Anti**symmetric field strength tensor.
- **Exterior** derivatives.
- **Spin 1** field.

$$\mathfrak{L}_{\mathrm{EM}} = -\frac{1}{c} J_q^{\mu} A_{\mu} - \frac{1}{4c^2}(A^{\nu,\mu} - A^{\mu,\nu})(A_{\nu,\mu} - A_{\mu,\nu})$$



Generalized EM Lagrange Density

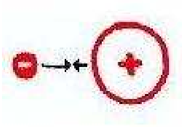
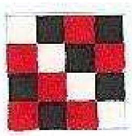
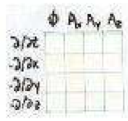
Extend the success of Maxwell.

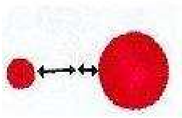
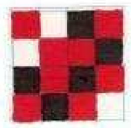
- Like electric charges **repel** and like mass charges **attract**.
- **A**symmetric field strength tensor tensor,

  the sum of **anti**symmetric and **symmetric** tensors.

- **Exterior** and **covariant** derivatives.
- **Spin 1** and **spin 2** fields.

$$\mathfrak{L}_{\mathrm{GEM}} = -\frac{1}{c}\,(J_q^{\mu} - J_m^{\mu})\,A_{\mu} - \frac{1}{2c^2}\,A^{\nu;\mu}\,A_{\nu;\mu}$$


Apply Euler-Lagrange

1. Start with the Euler-Lagrange equation, $\frac{\partial \mathfrak{L}}{\partial A^{\mu}} = (\frac{\partial \mathfrak{L}}{\partial A^{\mu,\nu}})^{,\nu}$, written without indices:

$$c\frac{\partial \mathfrak{L}}{\partial \phi} = c(\frac{\partial}{\partial t}(\frac{\partial \mathfrak{L}}{\partial(\frac{\partial \phi}{\partial t})}) - \frac{\partial}{\partial x}(\frac{\partial \mathfrak{L}}{\partial(-\frac{\partial \phi}{\partial x})}) - \frac{\partial}{\partial y}(\frac{\partial \mathfrak{L}}{\partial(-\frac{\partial \phi}{\partial y})}) - \frac{\partial}{\partial z}(\frac{\partial \mathfrak{L}}{\partial(-\frac{\partial \phi}{\partial z})}))$$

$$c\frac{\partial \mathfrak{L}}{\partial A_x} = c(\frac{\partial}{\partial t}(\frac{\partial \mathfrak{L}}{\partial(\frac{\partial A_x}{\partial t})}) - \frac{\partial}{\partial x}(\frac{\partial \mathfrak{L}}{\partial(-\frac{\partial A_x}{\partial x})}) - \frac{\partial}{\partial y}(\frac{\partial \mathfrak{L}}{\partial(-\frac{\partial A_x}{\partial y})}) - \frac{\partial}{\partial z}(\frac{\partial \mathfrak{L}}{\partial(-\frac{\partial A_x}{\partial z})}))$$

$$c\frac{\partial \mathfrak{L}}{\partial A_y} = c(\frac{\partial}{\partial t}(\frac{\partial \mathfrak{L}}{\partial(\frac{\partial A_y}{\partial t})}) - \frac{\partial}{\partial x}(\frac{\partial \mathfrak{L}}{\partial(-\frac{\partial A_y}{\partial x})}) - \frac{\partial}{\partial y}(\frac{\partial \mathfrak{L}}{\partial(-\frac{\partial \mathrm{Ay}}{\partial y})}) - \frac{\partial}{\partial z}(\frac{\partial \mathfrak{L}}{\partial(-\frac{\partial A_y}{\partial z})}))$$

$$c\frac{\partial \mathfrak{L}}{\partial A_z} = c(\frac{\partial}{\partial t}(\frac{\partial \mathfrak{L}}{\partial(\frac{\partial A_z}{\partial t})}) - \frac{\partial}{\partial x}(\frac{\partial \mathfrak{L}}{\partial(-\frac{\partial A_z}{\partial x})}) - \frac{\partial}{\partial y}(\frac{\partial \mathfrak{L}}{\partial(-\frac{\partial \mathrm{Az}}{\partial y})}) - \frac{\partial}{\partial z}(\frac{\partial \mathfrak{L}}{\partial(-\frac{\partial A_z}{\partial z})}))$$

2. Write out GEM Lagrange density without indices:

$$\begin{aligned}\mathfrak{L} = &-((\rho_q - \rho_m)\,\phi - (J_q^u - J_m^u)A^u) \\ &-\frac{1}{2}((\frac{\partial \phi}{c\partial t})^2 - (\frac{\partial \phi}{\partial x})^2 - (\frac{\partial \phi}{\partial y})^2 - (\frac{\partial \phi}{\partial z})^2 - (\frac{\partial A_x}{c\partial t})^2 + (\frac{\partial A_x}{\partial x})^2 + (\frac{\partial A_x}{\partial y})^2 + (\frac{\partial A_x}{\partial z})^2 \\ &-(\frac{\partial A_y}{c\partial t})^2 + (\frac{\partial A_y}{\partial x})^2 + (\frac{\partial A_y}{\partial y})^2 + (\frac{\partial A_y}{\partial z})^2 - (\frac{\partial A_z}{c\partial t})^2 + (\frac{\partial A_z}{\partial x})^2 + (\frac{\partial A_z}{\partial y})^2 + (\frac{\partial A_z}{\partial z})^2)\end{aligned}$$

3. Apply:

$$-(\rho_q - \rho_m)\,\phi = -\frac{\partial^2 \phi}{c\partial t^2} + c\frac{\partial^2 \phi}{\partial x^2} + c\frac{\partial^2 \phi}{\partial y^2} + c\frac{\partial^2 \phi}{\partial z^2}$$

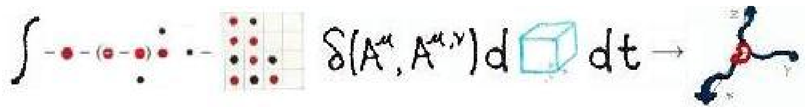
$$J_q^x - J_m^x = \frac{\partial^2 A_x}{c\partial t^2} - c\frac{\partial^2 A_x}{\partial x^2} - c\frac{\partial^2 A_x}{\partial y^2} - c\frac{\partial^2 A_x}{\partial z^2}$$

$$J_q^y - J_m^y = \frac{\partial^2 A_y}{c\partial t^2} - c\frac{\partial^2 A_y}{\partial x^2} - c\frac{\partial^2 A_y}{\partial y^2} - c\frac{\partial^2 A_y}{\partial z^2}$$

$$J_q^z - J_m^z = \frac{\partial^2 A_z}{c\partial t^2} - c\frac{\partial^2 A_z}{\partial x^2} - c\frac{\partial^2 A_z}{\partial y^2} - c\frac{\partial^2 A_z}{\partial z^2}$$

4. Executive summary:

   $J_q^\mu - J_m^\mu = \Box^2 A^\mu$



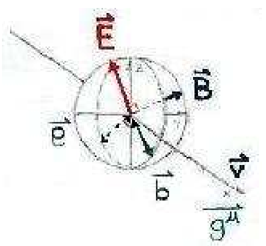
Classical Fields

The classical fields $\vec{E}$ and $\vec{B}$ together make up the antisymmetric tensor $(A^{\nu;\mu} - A^{\mu;\nu})$. Introduce three new fields, $\vec{e}$ and $\vec{b}$ which have EM counterparts, and a 4-vector field g^μ for the diagonal components of the symmetric tensor $(A^{\nu;\mu} + A^{\mu;\nu})$.

- $\vec{E} = -\frac{\partial \vec{A}}{\partial t} - c\vec{\nabla}\phi$ Electric field.
- $\vec{e} = \frac{\partial \vec{A}}{\partial t} - c\vec{\nabla}\phi - \Gamma_\sigma{}^{0i} A^\sigma$ Symmetric analog to electric field.
- $\vec{B} = c\vec{\nabla} \times \vec{A}$ Magnetic field.
- $\vec{b} = -\partial^i A^j - \partial^j A^i + \Gamma_\sigma{}^{ij} A^\sigma$ Symmetric analog to magnetic field.
- $g^\mu = A^{\mu,\mu} - \Gamma_\sigma{}^{\mu\mu} A^\sigma$ Diagonal of $A^{\nu;\mu}$.

3+3+3+3+4=16 fields total.

All three new fields transform differently than axial or polar vectors.



Classical Fields in Detail/public_html/quaternions/talks/General_em

1. Start with the asymmetric unified field strength tensor $A^{\nu;\mu}$ written as a matrix:

$$
\begin{array}{ccccc}
 & \nu = \phi & \nu = A_x & \nu = A_y & \nu = A_z \\
;\mu = \frac{\partial}{\partial t} & \frac{\partial \phi}{\partial t} - \Gamma_\sigma{}^{00} A^\sigma & \frac{\partial A_x}{\partial t} - \Gamma_\sigma{}^{10} A^\sigma & \frac{\partial A_y}{\partial t} - \Gamma_\sigma{}^{20} A^\sigma & \frac{\partial A_z}{\partial t} - \Gamma_\sigma{}^{30} A^\sigma \\
;\mu = -c\frac{\partial}{\partial x} & -c\frac{\partial \phi}{\partial x} - \Gamma_\sigma{}^{01} A^\sigma & -c\frac{\partial A_x}{\partial x} - \Gamma_\sigma{}^{11} A^\sigma & -c\frac{\partial A_y}{\partial x} - \Gamma_\sigma{}^{21} A^\sigma & -c\frac{\partial A_z}{\partial x} - \Gamma_\sigma{}^{31} A^\sigma \\
;\mu = -c\frac{\partial}{\partial y} & -c\frac{\partial \phi}{\partial y} - \Gamma_\sigma{}^{02} A^\sigma & -c\frac{\partial A_x}{\partial y} - \Gamma_\sigma{}^{12} A^\sigma & -c\frac{\partial A_y}{\partial y} - \Gamma_\sigma{}^{22} A^\sigma & -c\frac{\partial A_z}{\partial y} - \Gamma_\sigma{}^{32} A^\sigma \\
;\mu = -c\frac{\partial}{\partial z} & -c\frac{\partial \phi}{\partial z} - \Gamma_\sigma{}^{03} A^\sigma & -c\frac{\partial A_x}{\partial z} - \Gamma_\sigma{}^{13} A^\sigma & -c\frac{\partial A_y}{\partial z} - \Gamma_\sigma{}^{23} A^\sigma & -c\frac{\partial A_z}{\partial z} - \Gamma_\sigma{}^{33} A^\sigma
\end{array}
$$

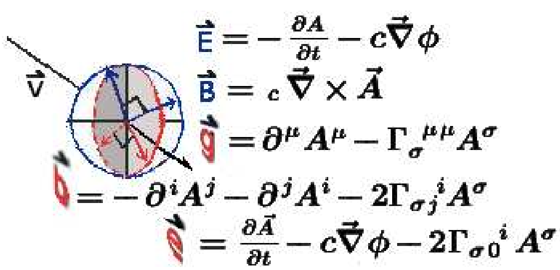
2. An antisymmetric and symmetric sum equal to $A^{\nu;\mu}$:

$$A^{\nu;\mu} - A^{\mu;\nu} = \begin{matrix} 0 & \frac{\partial A_x}{\partial t} + c\frac{\partial \phi}{\partial x} & \frac{\partial A_y}{\partial t} + c\frac{\partial \phi}{\partial y} & \frac{\partial A_z}{\partial t} + c\frac{\partial \phi}{\partial z} \\ -c\frac{\partial \phi}{\partial x} - \frac{\partial A_x}{\partial t} & 0 & -c\frac{\partial A_y}{\partial x} + c\frac{\partial A_x}{\partial y} & -c\frac{\partial A_z}{\partial x} + c\frac{\partial A_x}{\partial z} \\ -c\frac{\partial \phi}{\partial y} - \frac{\partial A_y}{\partial t} & -c\frac{\partial A_x}{\partial y} + c\frac{\partial A_y}{\partial x} & 0 & -c\frac{\partial A_z}{\partial y} + \frac{\partial A_y}{\partial z} \\ -c\frac{\partial \phi}{\partial z} - \frac{\partial A_z}{\partial t} & -c\frac{\partial A_x}{\partial z} + c\frac{\partial A_z}{\partial x} & -c\frac{\partial A_y}{\partial z} + c\frac{\partial A_z}{\partial y} & 0 \end{matrix}$$

$$A^{\nu;\mu} + A^{\mu;\nu} = \begin{matrix} 2(\frac{\partial \phi}{\partial t} - \Gamma_\sigma{}^{00}A^\sigma) & \frac{\partial A_x}{\partial t} - c\frac{\partial \phi}{\partial x} - 2\Gamma_\sigma{}^{01}A^\sigma & \frac{\partial A_y}{\partial t} - c\frac{\partial \phi}{\partial y} - 2\Gamma_\sigma{}^{20}A^\sigma & \frac{\partial A_z}{\partial t} - c\frac{\partial \phi}{\partial z} - 2\Gamma_\sigma{}^{30}A^\sigma \\ -c\frac{\partial \phi}{\partial x} + \frac{\partial A_x}{\partial t} - 2\Gamma_\sigma{}^{01}A^\sigma & 2(-c\frac{\partial A_x}{\partial x} - \Gamma_\sigma{}^{11}A^\sigma) & -c\frac{\partial A_y}{\partial x} - c\frac{\partial A_x}{\partial y} - 2\Gamma_\sigma{}^{21}A^\sigma & -c\frac{\partial A_z}{\partial x} - c\frac{\partial A_x}{\partial z} - 2\Gamma_\sigma{}^{31}A^\sigma \\ -c\frac{\partial \phi}{\partial y} + \frac{\partial A_y}{\partial t} - 2\Gamma_\sigma{}^{02}A^\sigma & -c\frac{\partial A_x}{\partial y} - c\frac{\partial A_y}{\partial x} - 2\Gamma_\sigma{}^{12}A^\sigma & 2(-c\frac{\partial A_y}{\partial y} - \Gamma_\sigma{}^{22}A^\sigma) & -c\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} - 2\Gamma_\sigma{}^{32}A^\sigma \\ -c\frac{\partial \phi}{\partial z} + \frac{\partial A_z}{\partial t} - 2\Gamma_\sigma{}^{03}A^\sigma & -c\frac{\partial A_x}{\partial z} - c\frac{\partial A_z}{\partial x} - 2\Gamma_\sigma{}^{13}A^\sigma & -c\frac{\partial A_y}{\partial z} - c\frac{\partial A_z}{\partial y} - 2\Gamma_\sigma{}^{23}A^\sigma & 2(-c\frac{\partial A_z}{\partial z} - \Gamma_\sigma{}^{33}A^\sigma) \end{matrix}$$

3. The asymmetric tensor written in terms of the five fields:

$$A^{\nu;\mu} = \begin{matrix} g_t & e_x - E_x & e_y - E_y & e_z - E_z \\ e_x + E_x & g_x & b_z - B_z & b_y + B_y \\ e_y + E_y & b_z + B_z & g_y & b_x - B_x \\ e_z + E_z & b_y - B_y & b_x + B_x & g_z \end{matrix}$$



Generalized Gauss' Law

Covers both gravity and EM.

Method: $\frac{1}{2}$ (EM law + gravitational analog) + diagonal terms = field equations.

$$\rho_q - \rho_m = \frac{1}{2}(\vec{\nabla} \cdot \vec{E} + \vec{\nabla} \cdot \vec{e}) + \frac{\partial g_t}{c\partial t}$$

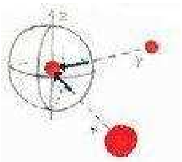
$$\rho_q - \rho_m = \frac{1}{2}(-\frac{\partial^2 A_x}{\partial x\,\partial t} - c\,\frac{\partial^2 \phi}{\partial x^2} - \frac{\partial^2 A_y}{\partial y\,\partial t} - c\,\frac{\partial^2 \phi}{\partial y^2} - \frac{\partial^2 A_z}{\partial z\,\partial t} - c\,\frac{\partial^2 \phi}{\partial z^2}$$

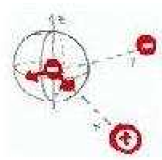
$$\frac{\partial^2 A_x}{\partial x\,\partial t} - c\,\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 A_y}{\partial y\,\partial t} - c\,\frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 A_z}{\partial z\,\partial t} - c\,\frac{\partial^2 \phi}{\partial z^2} + \frac{\partial^2 \phi}{c\partial t^2}$$

$$\rho_q - \rho_m = \frac{\partial^2 \phi}{c\partial t^2} - c\,\frac{\partial^2 \phi}{\partial x^2} - c\,\frac{\partial^2 \phi}{\partial y^2} - c\,\frac{\partial^2 \phi}{\partial z^2} = \Box^2 \phi$$

- Gauss' law results in the physical situation with no mass density and no change in the field $\vec{e}$.
- Newton's [relativistic] gravitational field equation results in the physical situation where there is no electric charge density and no divergence of the field $\vec{E}$.

Implications for forces: Newton's field law implies an attractive force for mass, while Gauss' law indicates like electric charges repulse.





Gen. Gauss Applied

Calculate the average general charge density for a proton in a 1 cm sphere.

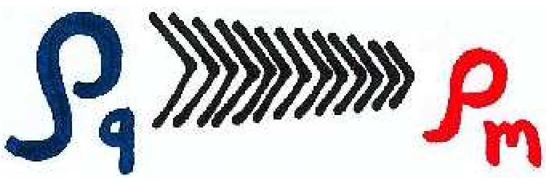
1. Electric charge density is charge/volume:

$$\rho_q = \frac{1.60 \times 10^{-19}\, C}{\frac{4\pi}{3}(0.0100 m)^3} = 3.82 \times 10^{-14}\, \frac{C}{m^3}$$

2. Mass charge density is $\sqrt{G}\, m_p$/volume:

$$\rho_m = \frac{\sqrt{6.67 \times 10^{-11}\, m^3\, kg^{-1}\, s^{-2}}\, 1.67 \times 10^{-19}\, C}{\frac{4\pi}{3}(0.0100 m)^3} = 3.25 \times 10^{-27}\, \frac{C}{m^3}$$

Thirteen orders of magnitude different!



The Exponential Metric

$$(\partial\tau)^2 = e^{-2\frac{GM}{c^2 R}}(\partial t)^2 - e^{+2\frac{GM}{c^2 R}}(\frac{\partial\vec{R}}{c})^2$$

This dynamic metric has exactly the same 10 Parameterized Post Newtonian (PPN) values as the Schwarzschild solution in general relativity:

$$\gamma = \beta = 1, \quad \xi = \alpha_1 = \alpha_2 = \alpha_3 = \zeta_1 = \zeta_2 = \zeta_3 = \zeta_4 = 0.$$

- Consistent with the weak equivalence principle ($m_{\text{inertial}} = m_{\text{passive}}$),

  The Nordtvedt effect:

  $\eta = 4\beta - \gamma - 3 - \frac{10}{3}\xi - \alpha_1 + \frac{2}{3}\alpha_2 - \frac{2}{3}\zeta_1 - \frac{1}{3}\zeta_2$, shown to be zero (Will, 8.9).

- Consistent with the strong equivalence principle ($m_{\text{passive}} = m_{\text{active}}$),

  Kreuzer's experiment:

  $\frac{m_{\text{active}}}{m_{\text{passive}}} = 1 + \frac{1}{2}\zeta_3 \frac{E_q}{m_p}$, shown to be 1 (Will 9.27).

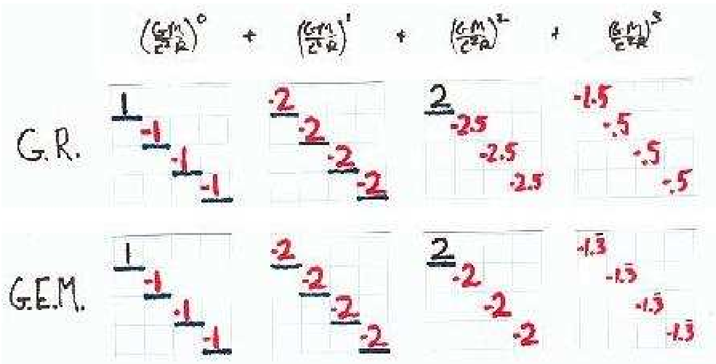
- Consistent with weak field tests, such as bending of light:

  $\delta\theta_{\max} = \frac{1}{2}(1+\gamma)1.75''$ (Will 7.23).

- Gravity waves travel at the speed of light. Since the metric is fully conservative, there cannot be a dipole due to conservation of momentum, indicating the quadrapole is the lowest mode of emission.

- Experimentally different for 2nd order PPN values.

- Exponential functions are the calling card of deep physics.



Metric Solves Gravitational Gauss' Law

1. Start with Gen. Gauss' law:

   $\rho_q - \rho_m = \frac{1}{2}(\vec{\nabla}\cdot\vec{E} + \vec{\nabla}\cdot\vec{e}) + \frac{\partial g_t}{c\partial t}$

2. Assume no electric charges or electromagnetic fields. Assume static:

   $\rho_m = -\vec{\nabla}\cdot\vec{e}$

3. Write out covariant derivative:

   $\rho_m = -\vec{\nabla}\cdot(\frac{\partial\vec{A}}{\partial t} - c\vec{\nabla}\phi - \Gamma_{\sigma}{}^{0i}A^{\sigma})$

4. Want everything to depend on the metric like in general relativity.

   Choose a constant potential s that $\frac{\partial\vec{A}}{\partial t} - c\vec{\nabla}\phi = 0$.

   $\rho_m = \vec{\nabla}\cdot\Gamma_{\sigma}{}^{0i}A^{\sigma}$ [Note: the divergence of the Christoffel symbol, cool!]

5. Write out the definition of the Christoffel symbol of the second kind:

   $\rho_m = \vec{\nabla}\cdot\frac{1}{2}g_{\sigma\beta}(g^{\beta i,0} + g^{0\beta,i} - g^{i0,\beta})A^{\sigma}$

6. Static, so $g^{\beta i,0} = 0$. Diagonal, so $g^{i0,\beta} = 0$.

   $\rho_m = \vec{\nabla}\cdot\frac{1}{2}g_{00}\,g^{00,i}A^{0}$

7. Write out in terms of the components:

$$\rho_m = \frac{\partial}{\partial x}\left(e^{2\frac{GM}{c^2\sqrt{x^2+y^2+z^2}}}\frac{\partial}{\partial x}e^{-2\frac{GM}{c^2\sqrt{x^2+y^2+z^2}}}\right)A^0/2 + \frac{\partial}{\partial y}\left(e^{2\frac{GM}{c^2\sqrt{x^2+y^2+z^2}}}\frac{\partial}{\partial y}e^{-2\frac{GM}{c^2\sqrt{x^2+y^2+z^2}}}\right)A^0/2$$
$$+ \frac{\partial}{\partial z}\left(e^{2\frac{GM}{c^2\sqrt{x^2+y^2+z^2}}}\frac{\partial}{\partial z}e^{-2\frac{GM}{c^2\sqrt{x^2+y^2+z^2}}}\right)A^0/2$$

8. This is the singular 1/R solution to the Poisson equation with canceling exponentials.

A physically-relevant, dynamic metric solves the GEM field equations.

$$\nabla e^{+\frac{1}{R}} \nabla e^{-\frac{1}{R}} = 0$$

A Background-Free Theory?

A dynamic metric **solves** the field equations,

∴ metric must be free enough to solve the differential equation!

versus

The 4-potential was varied, not the rank-2 metric

∴ the metric must be fixed!

Note: varying the metric field should apply to a rank 2 theory, not a rank 1 potential theory.

$$\delta A \begin{cases} \text{EM} \\ e^{-2\frac{m}{R}}dt^2 \end{cases} \underline{\text{vs}}\ \delta g \rightarrow \left(\frac{1-\frac{m}{2R}}{1+\frac{m}{2R}}\right)^2 dt^2$$

A Symmetry in Covariant Derivatives?

A covariant derivative ($A^{\nu;\mu} = A^{\nu,\mu} + \Gamma_\sigma{}^{\mu\nu} A^\sigma$) is the sum of two parts that are not tensors:

- $A^{\nu,\mu}$ is all about changes in the potential.
- $\Gamma_\sigma{}^{\mu\nu} A^\sigma$ is all about changes in the metric.

Until something is specified about either the potential or the metric, a covariant derivative could be **any** continuous combination of the change in potential and change in the metric.

Does that sound like a symmetry?

Is there a name already for this?

$$e^{-2\frac{m}{R}}dt^2 - e^{2\frac{m}{R}}dR^2 \ \underline{\text{or}}\ \nabla^2 A$$

Deriving the Exponential Metric

4D Wave Equation Vacuum Solution

1. Start with 4D wave equation, no source:

   $$\Box^2 A^\mu = 0$$

2. Guess a solution with similarities to previous:

   $$A^\mu = \frac{\sqrt{G}\,h}{c^2}\left((x^2+y^2+z^2-c^2t^2)^{-1}, 0, 0, 0\right) = \frac{\sqrt{G}\,h}{c^2}\left(\frac{1}{\sigma^2}, \vec{0}\right)$$

3. Take derivatives:

   $$\frac{\partial}{c\partial t}(x^2+y^2+z^2-c^2t^2)^{-1} = +2ct(x^2+y^2+z^2-c^2t^2)^{-2}$$

   $$\frac{\partial}{\partial x}(x^2+y^2+z^2-c^2t^2)^{-1} = -2x(x^2+y^2+z^2-c^2t^2)^{-2}$$

   $$\frac{\partial}{\partial y}(x^2+y^2+z^2-c^2t^2)^{-1} = -2y(x^2+y^2+z^2-c^2t^2)^{-2}$$

   $$\frac{\partial}{\partial z}(x^2+y^2+z^2-c^2t^2)^{-1} = -2z(x^2+y^2+z^2-c^2t^2)^{-2}$$

4. Take second derivatives:

   $$\frac{\partial}{c\partial t}(+2t\sigma^{-4}) = +2(x^2+y^2+z^2-c^2t^2)^{-2} + 8c^2t^2(x^2+y^2+z^2-c^2t^2)^{-4}$$

   $$\frac{\partial}{\partial x}(-2x\sigma^{-4}) = -2(x^2+y^2+z^2-c^2t^2)^{-2} + 8x^2(x^2+y^2+z^2-c^2t^2)^{-4}$$

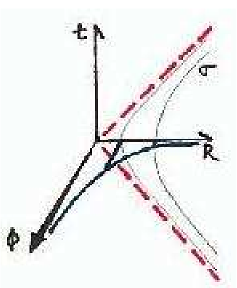
   $$\frac{\partial}{\partial y}(-2y\sigma^{-4}) = -2(x^2+y^2+z^2-c^2t^2)^{-2} + 8y^2(x^2+y^2+z^2-c^2t^2)^{-4}$$

   $$\frac{\partial}{\partial z}(-2z\sigma^{-4}) = -2(x^2+y^2+z^2-c^2t^2)^{-2} + 8z^2(x^2+y^2+z^2-c^2t^2)^{-4}$$

5. Sum:

   $$\frac{\partial^2 A_0}{c^2\partial t^2} - \frac{\partial^2 A_0}{\partial x^2} - \frac{\partial^2 A_0}{\partial y^2} - \frac{\partial^2 A_0}{\partial z^2} = 0 \qquad \text{QED}$$

- $x^2+y^2+z^2-c^2t^2 = 0$ Practical value: Singularity is the lightcone.
- $\vec{\nabla}\frac{1}{x^2+y^2+z^2-c^2t^2} \neq f\left(\frac{1}{R^2}\right)$ Practical problem: Derivative is not an inverse square law.



Normalized, Perturbation Solution

1. Start with 4D wave equation solution:

$$A^\nu = \frac{\sqrt{G}\,h}{c^2}\left(\frac{1}{x^2+y^2+z^2-c^2t^2}, \vec{0}\right) = \frac{\sqrt{G}\,h}{c^2}\left(\frac{1}{\sigma^2}, \vec{0}\right)$$

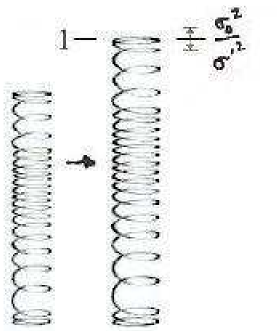
2. Normalize so that the magnitude of A^μ is equal to one:

$$A^\nu = \frac{A^\nu}{|A^\nu|} = \frac{c}{\sqrt{G}}\left(\frac{1}{\frac{x^2+y^2+z^2-c^2t^2}{\sigma^2}}, \vec{0}\right) = (1, \vec{0})$$

3. Perturb $x, y, z,$ and t linearly with a spring constant k:

$$A^\nu = \frac{A'^\nu}{|A^\nu|} = \frac{c}{\sqrt{G}}\left(\frac{1}{(\frac{1}{\sqrt{2}}+\frac{kx}{\sigma^2})^2+(\frac{1}{\sqrt{2}}+\frac{ky}{\sigma^2})^2+(\frac{1}{\sqrt{2}}+\frac{kz}{\sigma^2})^2-(\frac{1}{\sqrt{2}}+\frac{kct}{\sigma^2})^2}, \vec{0}\right)$$

$$= \frac{c}{\sqrt{G}}(\sim 1, \vec{0}) = \frac{c}{\sqrt{G}}\left(\frac{\sigma^2}{\sigma'^2}, \vec{0}\right)$$



Derivative of the Normalized, Perturbation Solution

1. Start with the normalized, perturbation solution:

$$A^\nu = \frac{A'^\nu}{|A^\nu|} = \frac{c}{\sqrt{G}}\left(\frac{1}{(\frac{1}{\sqrt{2}}+\frac{kx}{\sigma^2})^2+(\frac{1}{\sqrt{2}}+\frac{ky}{\sigma^2})^2+(\frac{1}{\sqrt{2}}+\frac{kz}{\sigma^2})^2-(\frac{1}{\sqrt{2}}+\frac{kct}{\sigma^2})^2}, \vec{0}\right) = \frac{c}{\sqrt{G}}\left(\frac{\sigma^2}{\sigma'^2}, \vec{0}\right)$$

2. Expand:

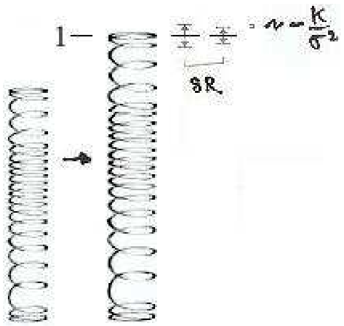
$$A^\nu = \frac{c}{\sqrt{G}}\left(\frac{1}{(\frac{1}{2}+\frac{kx}{\sqrt{2}\sigma^2}+\frac{k^2x^2}{\sigma^4})+(\frac{1}{2}+\frac{ky}{\sqrt{2}\sigma^2}+\frac{k^2y^2}{\sigma^4})+(\frac{1}{2}+\frac{kz}{\sqrt{2}\sigma^2}+\frac{k^2z^2}{\sigma^4})-(\frac{1}{2}+\frac{kct}{\sqrt{2}\sigma^2}+\frac{k^2c^2t^2}{\sigma^4})}, \vec{0}\right) = \frac{c}{\sqrt{G}}\left(\frac{\sigma_0^2}{\sigma'^2}, \vec{0}\right)$$

3. Take derivatives:

$$\frac{\partial A^\nu}{c\,\partial t} = \frac{c^2}{\sqrt{G}}\,\frac{\sigma_0^2}{\sigma'^4}\,k + O(k^2) \cong \frac{c^2}{\sqrt{G}}\,\frac{k}{\sigma^2} + O(k^2)$$

$$\frac{\partial A^\nu}{\partial R} = -\frac{c^2}{\sqrt{G}}\,\frac{\sigma_0^2}{\sigma'^4}\,k + O(k^2) \cong -\frac{c^2}{\sqrt{G}}\,\frac{k}{\sigma^2} + O(k^2)$$

- $\frac{1}{\sigma^2}$ An inverse square distance dependence.
- k A small number with units of distance.



Only Weak Gravity

A potential that only applies to gravity not EM will have a diagonal field strength tensor.

- The sign of the spring constant k does not effect solving the field equations.
- The sign of the spring constant k does change the derivative of the potential to first order in k.
- Therefore a potential that only has derivatives along the diagonal can be constructed from two potentials that differ by string constants that either constructively interfere to create a non-zero derivative, or destructively interfere to eliminate a derivative.

diagonal SHO $A^{\nu} = \frac{c^2}{\sqrt{G}}$

$$\Big(\frac{1}{(\frac{1}{\sqrt{2}}+\frac{kx}{\sigma^2})^2+(\frac{1}{\sqrt{2}}+\frac{ky}{\sigma^2})^2+(\frac{1}{\sqrt{2}}+\frac{kz}{\sigma^2})^2-(\frac{1}{\sqrt{2}}+\frac{kct}{\sigma^2})^2}+\frac{1}{(\frac{1}{\sqrt{2}}-\frac{kx}{\sigma^2})^2+(\frac{1}{\sqrt{2}}-\frac{ky}{\sigma^2})^2+(\frac{1}{\sqrt{2}}-\frac{kz}{\sigma^2})^2-(\frac{1}{\sqrt{2}}+\frac{kct}{\sigma^2})^2},$$

$$\frac{1}{(\frac{1}{\sqrt{2}}+\frac{kx}{\sigma^2})^2+(\frac{1}{\sqrt{2}}+\frac{ky}{\sigma^2})^2+(\frac{1}{\sqrt{2}}+\frac{kz}{\sigma^2})^2-(\frac{1}{\sqrt{2}}+\frac{kct}{\sigma^2})^2}+\frac{1}{(\frac{1}{\sqrt{2}}+\frac{kx}{\sigma^2})^2+(\frac{1}{\sqrt{2}}-\frac{ky}{\sigma^2})^2+(\frac{1}{\sqrt{2}}-\frac{kz}{\sigma^2})^2-(\frac{1}{\sqrt{2}}-\frac{kct}{\sigma^2})^2},$$

$$\frac{1}{(\frac{1}{\sqrt{2}}+\frac{kx}{\sigma^2})^2+(\frac{1}{\sqrt{2}}+\frac{ky}{\sigma^2})^2+(\frac{1}{\sqrt{2}}+\frac{kz}{\sigma^2})^2-(\frac{1}{\sqrt{2}}+\frac{kct}{\sigma^2})^2}+\frac{1}{(\frac{1}{\sqrt{2}}-\frac{kx}{\sigma^2})^2+(\frac{1}{\sqrt{2}}+\frac{ky}{\sigma^2})^2+(\frac{1}{\sqrt{2}}-\frac{kz}{\sigma^2})^2-(\frac{1}{\sqrt{2}}-\frac{kct}{\sigma^2})^2},$$

$$\frac{1}{(\frac{1}{\sqrt{2}}+\frac{kx}{\sigma^2})^2+(\frac{1}{\sqrt{2}}+\frac{ky}{\sigma^2})^2+(\frac{1}{\sqrt{2}}+\frac{kz}{\sigma^2})^2-(\frac{1}{\sqrt{2}}+\frac{kct}{\sigma^2})^2}+\frac{1}{(\frac{1}{\sqrt{2}}-\frac{kx}{\sigma^2})^2+(\frac{1}{\sqrt{2}}-\frac{ky}{\sigma^2})^2+(\frac{1}{\sqrt{2}}+\frac{kz}{\sigma^2})^2-(\frac{1}{\sqrt{2}}-\frac{kct}{\sigma^2})^2}\Big)$$

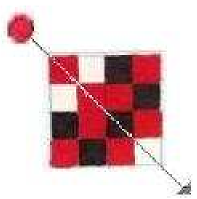
Notice the pattern for signs of k.

Take the contravariant derivative of this potential, which tosses in a minus sign.

Keep only the terms to first order in the spring constant k.

$$A^{\nu,\mu} \cong \frac{c^2}{\sqrt{G}}\ \frac{k}{\sigma 2}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}$$

This is the identity times $\frac{c^2}{\sqrt{G}}\ \frac{k}{\sigma 2}$, a simple end result that required much work.



Weak Field Approximation

1. Start from the gravitational force law:

$$F_G^\mu = -\frac{\sqrt{G}}{c}\, m U_\nu (A^{\nu;\mu} + A^{\mu;\nu}) = \frac{\partial m\, U^\mu}{\partial \tau}$$

2. Assume local covariant coordinates (; $\rightarrow$,):

$$F_G^\mu = -\frac{\sqrt{G}}{c}\, m U_\nu (A^{\nu,\mu} + A^{\mu,\nu}) = \frac{\partial m\, U^\mu}{\partial \tau}$$

3. Recall weak gravitational field strength tensor:

$$A^{\nu,\mu} \cong \frac{c^2}{\sqrt{G}}\, \frac{k}{\sigma 2} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

4. Check units of A^μ to the derivative of the normalized potential:

$$\sqrt{G}\; A^{\nu,\mu} \rightsquigarrow \frac{\sqrt{L^3}}{t\sqrt{m}}\; \frac{\sqrt{m}}{t\sqrt{L}} = \frac{L}{t^2}$$

$$c\, \frac{\partial \frac{A'^\nu}{|A^\nu|}}{\partial t} \rightsquigarrow \frac{L}{t}\; \frac{1}{t} = \frac{L}{t^2}$$

5. Substitute the normalized potential derivative into the force law. Expand the velocities. Assume spherical symmetry:

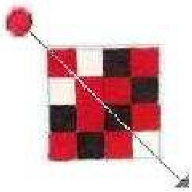
$$F_G^\mu = -\, m\, \frac{ck}{\sigma^2}\, (U_0, -\vec{U}) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \left(\frac{\partial m U_0}{\partial \tau}, \frac{\partial m \vec{U}}{\partial \tau}\right)$$

6. Contract the rank-1 velocity tensor with the rank-2 derivative of the potential:

$$F_G^\mu = m\, \frac{ck}{\sigma^2}\, (-U_0, \vec{U}) = \left(\frac{\partial m U_0}{\partial \tau}, \frac{\partial m \vec{U}}{\partial \tau}\right)$$

7. Substitute $c^2\tau^2$ for $-\sigma^2$:

$$F_G^\mu = m \frac{k}{c\tau^2}\, (U_0, -\vec{U}) = \left(\frac{\partial m U_0}{\partial \tau}, \frac{\partial m \vec{U}}{\partial \tau}\right)$$



Exact Solution

The gravitational force for the weak field is a first order differential equation that can be solved exactly.

1. Start from the gravitational force for a weak field:

   $$F_G^\mu = m \frac{k}{c\tau^2}\,(U_0, -\vec{U}) = (\frac{\partial m U_0}{\partial \tau}, \frac{\partial m \vec{U}}{\partial \tau})$$

2. Apply the chain rule to the cause terms. Assume $U_0 \frac{\partial m}{\partial \tau} = \vec{U}\,\frac{\partial m}{\partial \tau} = 0$.

   Collect terms on one side:

   $$(m\,\frac{\partial U_0}{\partial \tau} - m\,\frac{k}{c\tau^2}\,U_0, m\,\frac{\partial \vec{U}}{\partial \tau} + m\,\frac{k}{c\tau^2}\,\vec{U}) = 0$$

3. Assume the equivalence principle. Drop m:

   $$(\,\frac{\partial U_0}{\partial \tau} - \frac{k}{c\tau^2}\,U_0, \frac{\partial \vec{U}}{\partial \tau} + \frac{k}{c\tau^2}\,\vec{U}\,) = 0$$

4. Solve for velocity:

   $$(U_0, \vec{U}) = (c_0\,e^{-\frac{k}{c\tau}}, \vec{C}_{1-3}\,e^{+\frac{k}{c\tau}})$$

5. Contract the velocity solution:

   $$U^\mu\,U_\mu = c_0^2\,e^{-2\frac{k}{c\tau}} - \vec{C}_{1-3}\,e^{+2\frac{k}{c\tau}}$$

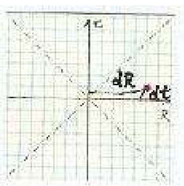
6. For flat spacetime ($k \to 0, \text{or}\, \tau \to \infty$), there are four constraints on the contracted velocity solution:

   $$U^\mu\,U_\mu = (c\,\frac{\partial t}{\partial \tau}, \frac{\partial \vec{R}}{\partial \tau})(c\,\frac{\partial t}{\partial \tau}, -\,\frac{\partial \vec{R}}{\partial \tau}) = \frac{c^2\,(\partial t)^2 - (\partial R)^2}{(\partial t)^2 - (\frac{\partial R}{c})^2} = c^2$$

   True if and only if: $\quad c_0^2 = c\,\frac{\partial t}{\partial \tau} = U_{0\,\text{flat}}, \quad \vec{C}_{1-3} = \frac{\partial \vec{R}}{\partial \tau} = \vec{U}_{\text{flat}}$

7. Substitute $c\,\frac{\partial t}{\partial \tau}$ for c_0^2, $\frac{\partial \vec{R}}{\partial \tau}$ for $\vec{C}_{1-3}$ into the contracted velocity solution. Multiply through by $(\frac{\partial \tau}{c})^2$:

   $$(\partial \tau)^2 = e^{-2\frac{k}{c\tau}}\,(\partial\,t)^2 - e^{+2\frac{k}{c\tau}}\,(\frac{\partial \vec{R}}{c})^2$$



Exact Solution Applied

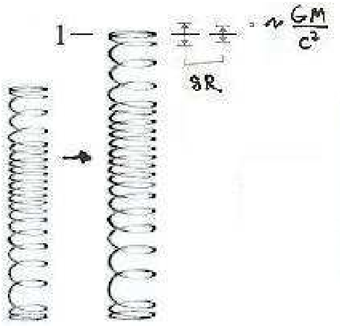
Apply to a weak, spherically symmetric, gravitational system.

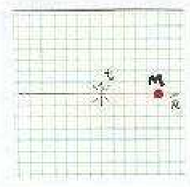
- $k = \frac{G M}{c^2} \rightsquigarrow \frac{L^3}{m\,t^2}\,m\,\frac{t^2}{L^2} = L$ Gravitational source spring constant.
- $\sigma^2 = R^2 - (c t)^2 = R'^2$ Static field approximated by R'.

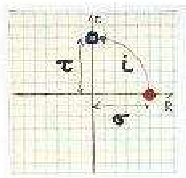
- $|\sigma| = |c\tau| = R$ $\qquad$ σ and $c\tau$ have the same magnitude.
- $(+i\sigma)^2 = (+c\tau)^2$ $\qquad$ To make a real metric, choose σ to be imaginary.

Plug into the exact solution:

$$(\partial\tau)^2 = e^{-2\frac{GM}{c^2 R}}(\partial t)^2 - e^{+2\frac{GM}{c^2 R}}\left(\frac{\partial\vec{R}}{c}\right)^2$$







Compare Schwarzschild to GEM Metrics

Write out the Taylor series expansion of the Schwarzschild and GEM metrics in isotropic coordinates to third order in $\frac{GM}{c^2 R}$.

1. Schwarzschild metric:

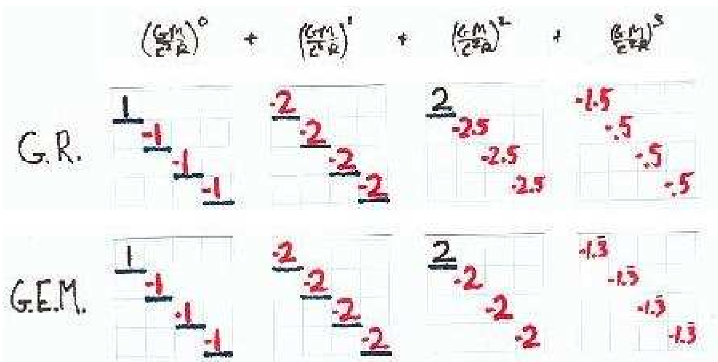
$$(\partial\tau)^2 = (\underline{1 - 2\frac{GM}{c^2 R} + 2\left(\frac{GM}{c^2 R}\right)^2} - \frac{3}{2}\left(\frac{GM}{c^2 R}\right)^3)(\partial t)^2 - (\underline{1 - 2\frac{GM}{c^2 R}} + \frac{3}{2}\left(\frac{GM}{c^2 R}\right)^2 + \frac{1}{2}\left(\frac{GM}{c^2 R}\right)^3)(\partial\vec{R})^2$$

2. GEM metric:

$$(\partial\tau)^2 = (\underline{1 - 2\frac{GM}{c^2 R} + 2\left(\frac{GM}{c^2 R}\right)^2} - \frac{4}{3}\left(\frac{GM}{c^2 R}\right)^3)(\partial t)^2 - (\underline{1 - 2\frac{GM}{c^2 R}} + 2\left(\frac{GM}{c^2 R}\right)^2 + \frac{4}{3}\left(\frac{GM}{c^2 R}\right)^3)\left(\frac{\partial\vec{R}}{c}\right)^2$$

Compare the two metrics:

- Identical for tested terms of Taylor series expansion.
- Different for higher order terms, so can be tested (not easy).
- GEM is more symmetric, uses exponentials!



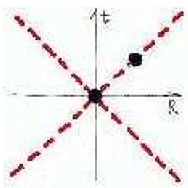
Classical Gravity and A New Constant Velocity Solution

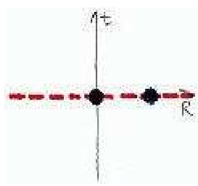
Lightlike Event Symmetry Breaking

Spacetime symmetry must be broken to go from the relativistic weak gravitational force to a classical force for both cause and effect.

Contrast the relativistic geometry of Minkowski spacetime with the geometry of Newtonian absolute space and time.

Minkowski Spacetime	Geometry	Newtonian Space and Time
True, Elegant	Utility	Accurate, Practical
$(\partial\tau)^2 = (dt)^2 - \frac{(dR)^2}{c^2}$	Interval	$\text{distance}^2 = dR^2 \neq f(t)$
$(U_0, \vec{U}) = (c\,\frac{\partial t}{\partial\tau}, \frac{\partial \vec{R}}{\partial\tau})$	Velocity	$(\mathbb{U}_0, \vec{\mathbb{U}}) \equiv (\frac{\partial t}{\partial\lvert R\rvert}, c\frac{\partial \vec{R}}{\partial\lvert R\rvert}) = (0, c\,\hat{R})$
$(\frac{\partial U_0}{\partial\tau}, \frac{\partial \vec{U}}{\partial\tau}) = (c\,\frac{\partial^2 t}{\partial\tau^2}, \frac{\partial^2 \vec{R}}{\partial\tau^2})$	Acceleration	$(\frac{\partial \mathbb{U}_0}{\partial\tau}, \frac{\partial \vec{\mathbb{U}}}{\partial\tau}) = (0, c^2\,\frac{\partial^2 \vec{R}}{\partial\lvert R\rvert^2})$





Newton's Law Derivation

1. Start from the gravitational force for a weak field:

$$F_G^\mu = m\,\frac{k}{c\tau^2}\,(U_0, -\vec{U}) = (\frac{\partial\, m U_0}{\partial\tau}, \frac{\partial\, m \vec{U}}{\partial\tau})$$

2. Apply the chain rule to the cause terms.

Assume $U_0\,\frac{\partial m}{\partial\tau} = \vec{U}\,\frac{\partial m}{\partial\tau} = 0$:

$$F_G^\mu = m\,\frac{k}{c\tau^2}\,(U_0, -\vec{U}) = (m\,\frac{\partial U_0}{\partial\tau}, m\,\frac{\partial \vec{U}}{\partial\tau})$$

3. Break spacetime symmetry:

- $(U_0, \vec{U}) \longrightarrow (\mathbb{U}_0, \vec{\mathbb{U}}) = (0, c\,\hat{R})$

- $(\frac{\partial U_0}{\partial\tau}, \frac{\partial \vec{U}}{\partial\tau}) \longrightarrow (0, c^2\,\frac{\partial^2 \vec{R}}{\partial\lvert R\rvert^2})$

$$F_G^\mu = m\,\frac{k}{\tau^2}\,(0, -\hat{R}) = (0, m\,c^2\,\frac{\partial^2 \vec{R}}{\partial\,|R|^2})$$

4. Assume the gravitational spring constant $(k = \frac{G\,M}{c^2})$:

$$F_G^\mu = (0, -\,\frac{G\,M\,m}{c^2\,\tau^2}\,\hat{R}) = (0, m\,c^2\,\frac{\partial^2 \vec{R}}{\partial\,|R|^2})$$

5. Substitute: σ^2 for $-\,c^2\,\tau^2$ in the cause term.

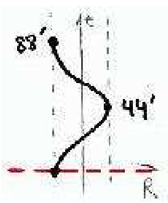
   Substitute: $-\,c^2\,(\frac{\partial}{\partial\tau})^2$ for $(\frac{\partial}{\partial|R|})^2 = (\frac{\partial}{\partial\sigma})^2$ in the effect term.

$$F_G^\mu = (0, \frac{G\,M\,m}{\sigma^2}\,\hat{R}) = (0, -\,m\,\frac{\partial^2 \vec{R}}{\partial\tau^2})$$

6. Assume the static field approximation: $\sigma^2 = R^2 - t^2 \cong R'^2$.

   Assume the low speed approximation: $\frac{\partial^2}{\partial\tau^2} \cong \frac{\partial^2}{\partial t^2}$:

$$F_G^\mu = (0, \frac{G\,M\,m}{R^2}\,\hat{R}) = (0, -\,m\,\frac{\partial^2 \vec{R}}{\partial t^2}) \qquad \text{QED}$$



Stable Constant Velocity Solutions

1. Start from the gravitational force for a weak field:

$$F_G^\mu = m\,\frac{k}{c\tau^2}\,(U_0, -\,\vec{U}) = (\frac{\partial\,m U_0}{\partial\,\tau}, \frac{\partial\,m\,\vec{U}}{\partial\,\tau})$$

2. Apply the chain rule to the cause terms.

   Assume $m\,\frac{\partial\,U_0}{\partial\tau} = m\,\frac{\partial\,\vec{U}}{\partial\tau} = 0$ (meaning assume velocity is constant):

$$F_G^\mu = m\,\frac{k}{c\tau^2}\,(U_0, -\,\vec{U}) = (U_0\,\frac{\partial\,m}{\partial\,\tau}, \vec{U}\,\frac{\partial\,m}{\partial\,\tau})$$

3. Assume classical and conservative: $U_0 \longrightarrow 0$.

$$F_G^\mu = m\,\frac{k}{\tau^2}\,(0, -\,\vec{U}) = (0, \frac{\partial\,m}{\partial\,\tau}\,\vec{U})$$

4. Assume the gravitational spring constant $(k = \frac{G\,M}{c^2})$:

$$F_G^\mu = (0, -\,\frac{G\,M\,m}{c^2\,\tau^2}\,\vec{U}) = (0, \frac{\partial\,m}{\partial\,\tau}\,\vec{U})$$

5. Collect terms on one side:

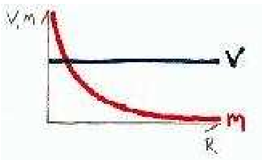
   $$(c\frac{\partial m}{\partial \tau}+\frac{GMm}{c^2\tau^2})(0,\vec{U})=0$$

6. Solve for m:

   $$m=m_o e^{\frac{GM}{c^3\tau}}$$

7. Substitute: R for $c\tau$ which depends on exactly the same assumptions used in the metric derivation (static field, $|\sigma|=|\tau|$=R, and sigma is imaginary):

   $$m=m_0 e^{\frac{GM}{c^2 R}}$$



From a Relativistic 4-Force to the Constant Velocity Solution

Start from a general 4-force law:

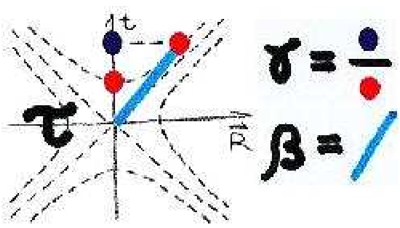
$$F^\mu=\frac{d\rho V^\mu}{d\tau}$$

- 4-velocity, V^μ.

  $$V^\mu=c(\gamma,\gamma\beta)$$

- Spacetime interval, τ.

  $$(d\tau)^2=(dt)^2-(d\vec{R}\cdot d\vec{R})/c^2$$



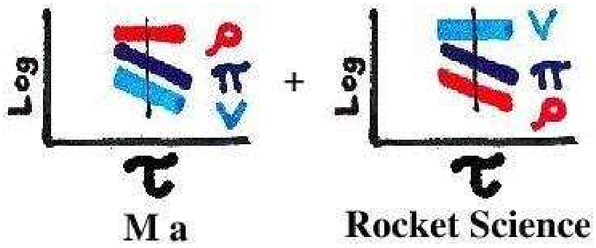
The Chain Rule for a 4-Force

The change in momentum with respect to spacetime equals

the change in 4-velocity with respect to the interval **plus**

the change in mass with respect to the spacetime interval.

$$\frac{d\rho V^\mu}{d\tau}=\rho\frac{dV^\mu}{d\tau}+V^\mu\frac{d\rho}{d\tau}$$



Relativistic to Classical Force

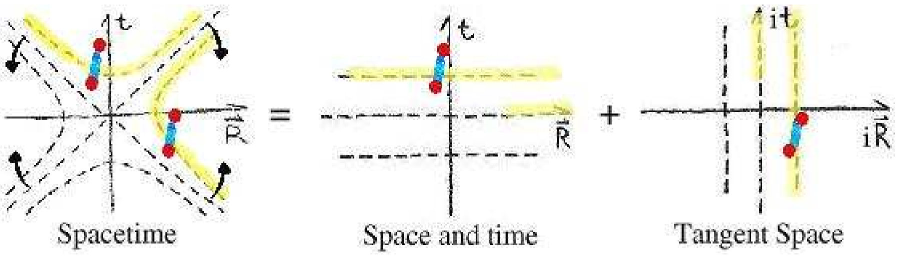
- 4-velocity becomes a 3-vector:

  $V^{\mu} \rightarrow \vec{V}$

- Change in **spacetime** becomes either

  $d\tau \rightarrow dt$, a change in **time** for timelike events in Newtonian space and time or

  $d\tau \rightarrow d\,|R/c|$, a change in **space** for spacelike events in the complex tangent space.



The **completely arbitrary** location of the spacetime origin means in the transformation from Minkowski spacetime to Newtonian space and time, the slope of some worldline for massive particles are real, while others are undefined in space and time unless the complex tangent space is included.

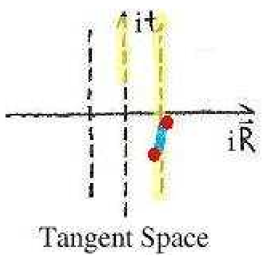
The Complex Tangent Space

The distance squared, $(d\tau)^2$, must be a real, positive definite number.

For spacelike separated events, $dR/c > dt$.

$$(d\tau)^2 = (i\,dt)^2 - (i\,dR/c)^2$$

is a real, positive definite number.



One cannot travel a distance iR, or measure a time it, however

gamma, beta, and tau are all real numbers!

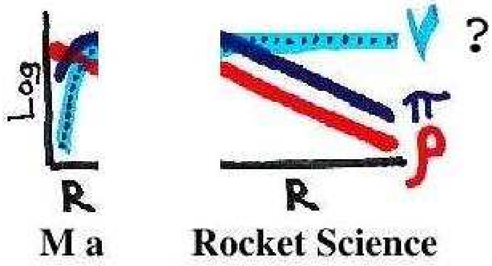
A New Direction for the Force of Gravity Hypothesis

The cause of gravity is the same, $-\frac{GM\rho}{R^2}$.

There is a new effect in the direction of the velocity vector in the complex tangent space.

$$\vec{F} = -\frac{GM\rho}{R^2}(\hat{R} + \hat{V}) = \rho\frac{V^2}{|R|}\hat{R} + \frac{d\rho}{d|R/c|}\vec{V}$$

The new terms are well-formed as far as units and vectors are concerned.



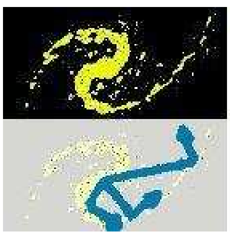
The Thin Disc Galaxy NGC3198

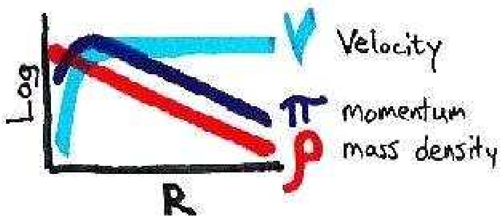
Not a point source, extending 200,000 light years across.

Outside the core region

- Velocity is **constant** at 150,000 m/s.
- Mass density decreases **exponentially**,

  mass/area = 37 Exp (-R'/2.23') solar masses/pc^2.
- Total mass is 1.0 x 10^{40} kg.





The BIG Newtonian Problems

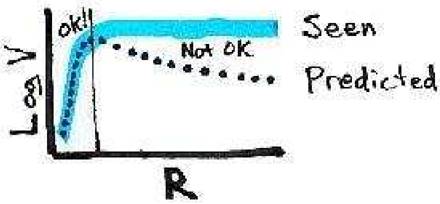
1. Unstable.

   Disc galaxies should have collapsed by now.

2. Velocity profile cannot remain flat.

   Newtonian constant V solution needs $M(R) = f(\frac{1}{R^2})$.

   An exponential drop at large radii is too fast for V to be constant.



Alternative hypotheses:

- Dark Matter.
- Modification of Newtonian Dynamics (MOND).

Test of Hypothesis for NGC3198

1. Start from Newtonian cause equal to rocket science effect:

   $$-\frac{GM\rho}{R^2}\,\hat{V}\ ?=?\ \frac{d\rho}{d|\vec{R}/c|}\,\vec{V}$$

2. Collect on one side to form a first order differential equation:

   $$\left(\frac{d\rho}{dR} + \frac{GM\rho}{cVR^2}\right)\hat{V}\ = 0$$

3. Solve for the mass density:

   $$\rho = k\,\mathrm{Exp}\left(\frac{GM}{cVR}\right)$$

4. Plug in values for the galaxy NGC3198:

   $$G = 6.7x10^{-11}\,m^3\,\mathrm{kg}^{-1}\,s^{-2}, M = 1.0x10^{40}\,\mathrm{kg},$$

   $$c = 3.0x10^8\,m/s, V = 1.5x10^5\,m/s, 3.1x10^{16}\,m/'$$

   $$\rho = k\,\mathrm{Exp}\,(.48'/R')$$

5. For large R:

   $$\rho = k\,\mathrm{Exp}\,(.48'/R') \approx 1 + k\,\mathrm{Exp}\,(-R'/2.1')$$

6. Note the similarity to the given mass density, ρ = 37 Exp (-R'/2.23').

Conclusion: The new "rocket science" effect of gravity deserves a detailed numerical study to see how well it agrees with all the data from the center outward.

Quantum Gravity

Momentum of Classical EM

1. Start with the EM Lagrange density written without indices.

$$\mathfrak{L}_{\text{EM}} = -\frac{\rho_m}{\gamma} - \frac{1}{c}J^\mu A_\mu - \frac{1}{4c^2}(A^{\mu.\nu} - A^{\nu,\mu})(A_{\mu,\nu} - A_{\nu,\mu})$$

$$= -\rho_m\sqrt{1 - (\frac{\partial x}{c\,\partial t})^2 - (\frac{\partial y}{c\,\partial t})^2 - (\frac{\partial z}{c\,\partial t})^2} - \rho_q(c\phi - \frac{\partial x}{\partial t}A_x - \frac{\partial x}{\partial t}A_y - \frac{\partial z}{\partial t}A_z)$$

$$-\frac{1}{2}((\quad -(\frac{\partial\phi}{\partial x})^2 - (\frac{\partial\phi}{\partial y})^2 - (\frac{\partial\phi}{\partial z})^2 - (\frac{\partial A_x}{c\,\partial t})^2 \qquad + (\frac{\partial A_x}{\partial y})^2 + (\frac{\partial A_x}{\partial z})^2$$

$$-(\frac{\partial A_y}{c\,\partial t})^2 + (\frac{\partial A_y}{\partial x})^2 \qquad + (\frac{\partial A_y}{\partial z})^2 - (\frac{\partial A_z}{c\,\partial t})^2 + (\frac{\partial A_z}{\partial x})^2 + (\frac{\partial A_z}{\partial y})^2$$

$$-2\frac{\partial A_x}{c\,\partial t}\frac{\partial\phi}{\partial x} - 2\frac{\partial A_y}{c\,\partial t}\frac{\partial\phi}{\partial y} - 2\frac{\partial A_z}{c\,\partial t}\frac{\partial\phi}{\partial z} - 2\frac{\partial A_y}{\partial z}\frac{\partial A_z}{\partial y} - 2\frac{\partial A_z}{\partial x}\frac{\partial A_x}{\partial z} - 2\frac{\partial A_x}{\partial y}\frac{\partial A_y}{\partial x})$$

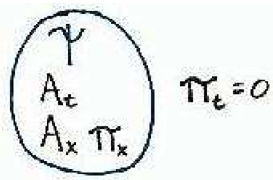
2. Calculate momentum:

$$\pi^\mu = h\sqrt{G}\frac{\partial\mathfrak{L}}{\partial\frac{\partial A^\mu}{c\partial t}} = h\sqrt{G}\,(0, \frac{\partial A_x}{c\partial t} + \frac{\partial\phi}{\partial x}, \frac{\partial A_y}{c\partial t} + \frac{\partial\phi}{\partial y}, \frac{\partial A_z}{c\partial t} + \frac{\partial\phi}{\partial z})$$

Energy-momentum vector.

3. Momentum cannot be made into an operator:

$[A_t, \pi_t]|\psi\rangle = [A_t, 0]|\psi\rangle = 0$ Energy commutes with its conjugate operator.



Quantizing EM by Fixing the Lorenz Gauge

Fix the Lorenz gauge in the EM Lagrange density.

1. Start with the Gupta-Bleuler Lagrange density written without indices:

$$\mathfrak{L}_{G-B} = -\frac{\rho_m}{\gamma} - \frac{1}{c}J^\mu A_\mu - \frac{1}{2c^2}(A^\mu{}_{,\mu})^2 - \frac{1}{4c^2}(A^{\mu.\nu} - A^{\nu,\mu})(A_{\mu,\nu} - A_{\nu,\mu})$$

$$= -\rho_m\sqrt{1 - (\frac{\partial x}{c\,\partial t})^2 - (\frac{\partial y}{c\,\partial t})^2 - (\frac{\partial z}{c\,\partial t})^2} - \rho_q(c\phi - \frac{\partial x}{\partial t}A_x - \frac{\partial x}{\partial t}A_y - \frac{\partial z}{\partial t}A_z)$$

$$-\frac{1}{2}((\frac{\partial\phi}{c\,\partial t})^2 - (\frac{\partial\phi}{\partial x})^2 - (\frac{\partial\phi}{\partial y})^2 - (\frac{\partial\phi}{\partial z})^2 - (\frac{\partial A_x}{c\,\partial t})^2 + (\frac{\partial A_x}{\partial x})^2 + (\frac{\partial A_x}{\partial y})^2 + (\frac{\partial A_x}{\partial z})^2$$

$$-(\frac{\partial A_y}{c\,\partial t})^2 + (\frac{\partial A_y}{\partial x})^2 + (\frac{\partial A_y}{\partial y})^2 + (\frac{\partial A_y}{\partial z})^2 - (\frac{\partial A_z}{c\,\partial t})^2 + (\frac{\partial A_z}{\partial x})^2 + (\frac{\partial A_z}{\partial y})^2 + (\frac{\partial A_z}{\partial z})^2$$

$$-2\frac{\partial A_x}{c\,\partial t}\frac{\partial\phi}{\partial x} - 2\frac{\partial A_y}{c\,\partial t}\frac{\partial\phi}{\partial y} - 2\frac{\partial A_z}{c\,\partial t}\frac{\partial\phi}{\partial z} - 2\frac{\partial A_y}{\partial z}\frac{\partial A_z}{\partial y} - 2\frac{\partial A_z}{\partial x}\frac{\partial A_x}{\partial z} - 2\frac{\partial A_x}{\partial y}\frac{\partial A_y}{\partial x})$$

$$+2\frac{\partial\phi}{c\,\partial t}\frac{\partial A_x}{\partial x} + 2\frac{\partial\phi}{c\,\partial t}\frac{\partial A_y}{\partial y} + 2\frac{\partial\phi}{c\,\partial t}\frac{\partial A_z}{\partial z} + 2\frac{\partial A_x}{\partial x}\frac{\partial A_y}{\partial y} + 2\frac{\partial A_x}{\partial x}\frac{\partial A_z}{\partial z} + 2\frac{\partial A_y}{\partial y}\frac{\partial A_z}{\partial z})$$

2. Calculate momentum:

$$\pi^{\mu} = h\sqrt{G}\frac{\partial \mathfrak{L}}{\partial \frac{\partial A^{\mu}}{c\partial t}} = h\sqrt{G}\,(-\frac{\partial \phi}{c\partial t} - \vec{\nabla}\cdot\vec{A}, \frac{\partial A_x}{c\partial t} + \frac{\partial \phi}{\partial x}, \frac{\partial A_y}{c\partial t} + \frac{\partial \phi}{\partial y}, \frac{\partial A_z}{c\partial t} + \frac{\partial \phi}{\partial z})$$

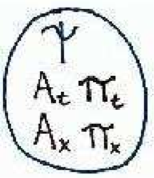
Energy-momentum vector.

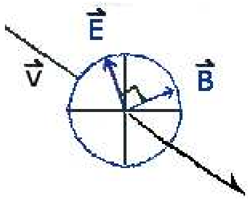
3. Momentum can be made into an operator:

Using the Euler-Lagrange equation [not shown], the equations of motion are identical to those of $\mathfrak{L}_{\mathrm{GEM}}$!

$J^{\mu} = \Box^2 A^{\mu}$

Reference: "Theory of longitudinal photons in quantum electrodynamics", Suraj N. Gupta, Proc. Phys. Soc. 63:681-691, 1950.

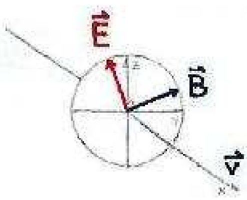




Gupta/Bleuler Quantization Method

Results of quantization method:

- Four modes of transmission:
    1. Two transverse spin 1 modes of transmission.
    2. One spin 1 longitudinal mode.
    3. One spin 1 scalar mode.
- Transverse waves are photons for EM.
- The supplementary condition is imposed to eliminate scalar and longitudinal modes for photons as real particles. There can be no scalar or longitudinal modes for a spin 1 4D wave field.



Momentum of GEM Lagrange Density

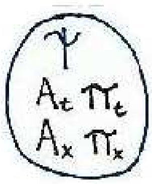
1. Start with the GEM Lagrange density written without indices:

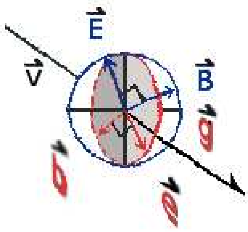
$$\mathfrak{L} = -\rho_m(\sqrt{1 - (\frac{\partial x}{c\partial t})^2 - (\frac{\partial y}{c\partial t})^2 - (\frac{\partial z}{c\partial t})^2} - (\rho_q - \sqrt{G}\,\rho_m)(c\phi - \frac{\partial x}{\partial t}A_x - \frac{\partial x}{\partial t}A_y - \frac{\partial z}{\partial t}A_z)$$
$$-\frac{1}{2}((\frac{\partial \phi}{c\partial t})^2 - (\frac{\partial \phi}{\partial x})^2 - (\frac{\partial \phi}{\partial y})^2 - (\frac{\partial \phi}{\partial z})^2 - (\frac{\partial A_x}{c\partial t})^2 + (\frac{\partial A_x}{\partial x})^2 + (\frac{\partial A_x}{\partial y})^2 + (\frac{\partial A_x}{\partial z})^2$$
$$-(\frac{\partial A_y}{c\partial t})^2 + (\frac{\partial A_y}{\partial x})^2 + (\frac{\partial A_y}{\partial y})^2 + (\frac{\partial A_y}{\partial z})^2 - (\frac{\partial A_z}{c\partial t})^2 + (\frac{\partial A_z}{\partial x})^2 + (\frac{\partial A_z}{\partial y})^2 + (\frac{\partial A_z}{\partial z})^2)$$

2. Calculate momentum:

$$\pi^{\mu} = h\sqrt{G}\,\frac{\partial \mathfrak{L}}{\partial \frac{\partial A^{\mu}}{c\partial t}} = h\sqrt{G}\,(-\frac{\partial \phi}{c\partial t}, \frac{\partial A_x}{c\partial t}, \frac{\partial A_y}{c\partial t}, \frac{\partial A_z}{c\partial t})$$

3. Momentum can be made into an operator.





GEM Quantization

- Four modes of transmission:
    1. Two spin 1 transverse modes.
    2. One spin 2 longitudinal mode.
    3. One spin 2 scalar mode.
- Transverse modes are photons for EM.
- Longitudinal and scalar modes are gravitons of gravity traveling at the speed of light, generated by a symmetric rank-2 field strength tensor.
- General relativity predicts transverse waves, not scalar or longitudinal ones. The LIGO experiment to detect gravitational waves will be looking for transverse gravitational waves. GEM predicts the polarization will not be transverse.