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Geometry + 4-potential = Unified Field Theory

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Gravity is the study of geometry. Light is the study of potentials. A unified field theory would have to show how geometry and potentials could share the work of describing gravity and light. There is a long list of criteria that must be satisfied to have a reasonable hypothesis, from recreating the Maxwell equations, to passing the classical tests of gravity, to demonstrating consistency with the equivalence principle, and working well with quantum mechanics. This essay works through many of the common objections.

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Geometry without a potential is like a bed without a lover. The Riemann curvature tensor, with its divergence of two connections, is exclusively about geometry and all about the bed sheet. Newton's scalar potential theory was the first math to reach and direct the motion of the stars. It is only about the scalar potential. Unfortunately, it is too small, being inconsistent with special relativity. I will try to construct a unified field theory for gravity and electromagnetism as a compromise between Newton and Einstein, the potential and the metric, in a way that will get along with quantum mechanics. My guiding principle is provided by Goldielocks, who might find a scalar theory too small, a rank 2 theory too large, so perhaps a rank 1 proposal will be just right.

I honestly love Newton's potential theory. It is still in use today by rocket scientists who do not put an atomic clock onboard their ship. It gets half the answer right about light bending around the Sun. When a theory comes up short, we can either discard it or figure out the simplest way to lend a hand. A gravitational theory with a thousand potentials instead of one will be able to match every experimental test of gravity. Use Occam's razor: a 4-potential should be more than adequate to match how gravity makes the measurement of time get a little smaller, while the measurement of 3-space gets a little larger.

I am torn between two lovers, Newton and Einstein, feeling like a fool. Thugs from Ulm will insist that gravity must be a metric theory. They have the experimental tests of the equivalence principle to prove it. They punch home the fact that the way to take the derivative of a connection that transforms like a tensor is through the Riemann curvature tensor. Drop the Ricci scalar into an action, vary it with respect to the metric, and out from the heavens flies Einstein's field equations.

What's wrong with that? A metric theory isn't silly at all. One must be able to express gravity in terms of a metric. Based on my respect for Newton, I wonder if it is possible to find a compromise between a larger 4-potential and a metric theory?

When we were young, we would write a covariant tensor as A_ν. The differential ∂_μ also transforms like a tensor. When we bring these two together, the 4-derivative of a 4-potential, $\partial_\mu A_\nu$, the result does not transform like a tensor. The reason is that as we move around a manifold, the manifold - not the potential - might change. A means of accounting for a changing surface must be made. Here is the definition of a covariant derivative all students of gravity learn:

$$\nabla_\mu A_\nu = \partial_\mu A_\nu - \Gamma^\sigma{}_{\mu\nu} A_\sigma$$

Can you spot the symmetry and identify the group implied by this definition? Imagine we make a measure of one of these terms, say $\nabla_0 A_0$, and it happens to be 1.007. If one worked in flat Euclidean spacetime, the connection would be zero everywhere, and everything would come from the change in the potential, $\partial_0 A_0$. One could also decide to use a constant potential, so the dynamic metric's connection would account for all the change seen, $-\Gamma^\sigma{}_{00} A_\sigma$. One has the ability to continuously change the metric and thus the connection so long as there is a corresponding change in the potential which leaves the resulting covariant derivative invariant. This sounds like the group Diff(M) of all diffeomorphisms of a 4D spacetime with the additional constraint that there are changes in the 4-potential such that the covariant derivative is unaltered.

Born background free, as free as general relativity, one must find a differential equation whose solution will dictate the terms of the dynamic metric. That is what the derivative of the connection in the Riemann curvature tensor does: there are second derivatives of the metric whose solutions under simple circumstances can be found. I have chosen to study the simplest vacuum 4D wave equation:

$$\square^2 A_\mu = 0$$

It is vital to note that I did not write the D'Alembertian operator, which would have been a box without the 2. Instead this is a covariant derivative acting on a contravariant derivative acting on the 4-potential. The first derivative will bring in a connection, and the second derivative will take the derivative of the connection, resulting in a second order differential equation of the metric, precisely what is needed to be background free. Can we find interesting combinations of metrics and potentials that solve this differential equation and is consistent with all tests of gravity to date?

Say we used a constant potential, where all the second derivatives were zero. Make the problem simple: a static, spherically symmetric, and non-rotating mass. For those skilled in the arts of differential geometry, it should be straightforward to show that the divergence of the connection of the exponential metric (below) is a non-trivial, entirely metric solution to the 4D wave equation. Compare the exponential metric in isotropic coordinates:

$$d\tau^2 = \exp(-2\frac{GM}{c^2R})dt^2 - \frac{1}{c^2}(\exp(2\frac{GM}{c^2R})(dx^2 + dy^2 + dz^2)$$

a nicely matched pair of exponentials, with the Schwarzschild solution in isotropic coordinates:

$$d\tau^2 = (\frac{1 - \frac{GM}{2c^2R}}{1 + \frac{GM}{2c^2R}})^2 dt^2 - \frac{1}{c^2}(1 + \frac{GM}{2c^2R})^4(dx^2 + dy^2 + dz^2)$$

which is inelegant enough to rarely be seen in books on general relativity. Theorist prefer the Schwarschild coordinates while experimentalists must work with isotropic ones. Beauty may be in the eye of the beholder, but an exponential is the calling card of a deep insight into physics.

Either metric satisfies all tests of the equivalence principle because the solution is written as a metric. Either metric satisfies all tests of the weak field because their Taylor series is the same to the terms tested. Either metric satisfies all strong field tests because it is entirely about a metric, so there is no other field to store energy or momentum. For an isolated system, the lowest mode of emission is the quadrapole moment. The metrics differ in second order effects by twenty percent in how much light is bent around the Sun, so it is a shame no one has been funded to get the data.

The 4D wave equation has been quantized, and written up in most books on quantum field theory, in the section on relativistic quantization of the Maxwell equations. Two of the modes of emission are the transverse spin 1 fields of light. That is no surprise. The scalar and longitudinal modes are banished to a virtual state using a "supplementary condition" because the scalar mode would allow negative probabilities, a no-no. That is the way it is for a spin 1 field theory where like charges repel. The field strength reducible asymmetric tensor $\nabla_\mu A_\nu$ for this proposal can be split in two: an irreducible antisymmetric rank 2 tensor to do the work of electromagnetism with a spin 1 field so like electric charges repel, and an irreducible symmetric rank 2 tensor to do the work of gravity with a spin 2 field so like mass charges attract. Gravity couples to the 4-momentum, not the rank 2 stress-energy tensor. All forms of energy go into both sources, except one: the energy of a gravitational field. To be consistent with electromagnetism, gravity fields do not gravitate. Should a gravity wave ever be detected and measured along six axes, the polarization of that wave will be transverse if general relativity is correct, but not if this unified field proposal is accurate. Such data will be hard to get, but the difference would be unambiguous.

The speed of gravity is the speed of light, and so its field strength tensor must be gauge invariant. The field strength tensor $\nabla_\mu A_\nu$ is only gauge invariant if its trace happens to be zero. That is where the massless graviton lives. When the trace is not zero, then the scalar field formed from the trace of $\nabla_\mu A_\nu$ will break the U(1) symmetry of electromagnetism. The Higgs particle is unnecessary. There is a quantum expression of the equivalence principle, a link between the spin 2 particle ($\nabla_\mu A_\nu$ when $\mathrm{tr}(\nabla_\mu A_\nu) = 0$) that mediates gravity and the scalar field needed to establish inertia ($\mathrm{tr}(\nabla_\mu A_\nu) \neq 0$).

There is an important benefit to splitting the load for describing gravity between the connection and the changes in the potential. By using Riemann normal coordinates, an arbitrary point in spacetime can have a connection equal to zero. For that point, the energy will be zero. That has remained a technical problem for people trying to quantize general relativity. For this proposal, the energy contributed by the connection could be zero, but that contributed by the potential would be non-zero. Localized energy is a good thing.

Einstein had a great respect for Newton's towering body of work. He might have appreciated this compromise between geometry and potentials which allows light to lay down with gravity in the same equation.