Unifying Gravity and EM by Analogies to EM

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Short Description:

Investigate an old hypothesis, that gravity is similar to EM. Clone a Lagrange density for gravity from EM. A problem has been the distance dependence. Impress friends by deriving Newton's law of gravity using perturbations of a normalized potential. The mundane chain rule may eliminate the need for dark matter and energy. The subtle underlying idea will be discussed while Grand Marnier chocolate truffles are served.

Skeptics welcome.

Day 3: Table of Contents

Outline for Day 3

Must Do Physics.

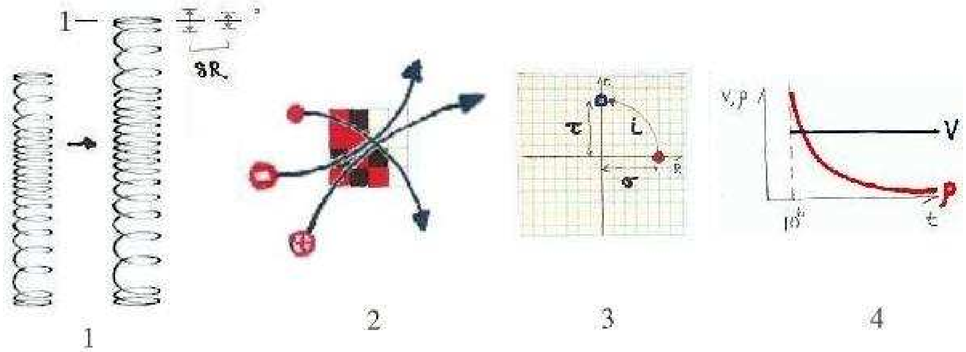
1. Field equation solutions.

2. Stresses, forces, and geodesics.

3. Relativistic gravitational force.

4. Classical gravitational force.

   Must Do Physics Done.



The Big Retro/Modern Picture

Retro Problems:

1. Maxwell Equations (and only EM) in the Lorenz gauge (1870?): $J^\nu = \Box^2 A^\nu$

2. Gravity is an inverse square law, so the potential must be $\frac{1}{\text{distance}}$ (~ 1800).

3. Toss out average derivatives, add in the gauge (1950):

   $\mathfrak{L}_{G-B} = -\frac{1}{\gamma}\rho_m - J^\mu A_\mu - \frac{1}{2c^2}(\nabla_\mu A^\mu)^2 - \frac{1}{4c^2}(\nabla^\mu A^\nu - \nabla^\nu A^\mu)(\nabla_\mu A_\nu - \nabla_\nu A_\mu)$

Modern solutions:

1. 4D Wave equation requires a 4D wave, 2 for EM, 2 for gravity (2000).

2. Gravity is weak, so perturbation theory is required $\phi \rightarrow \frac{\phi_0 + kx}{|\phi_0|}$ (2002).

3. A complete picture requires nothing be tossed or added (2002):

   $\mathfrak{L}_{\text{GEM}} = -\frac{1}{\gamma}\rho_m - (J_q^\mu - J_m^\mu) A_\mu - \frac{1}{2c^2}\nabla^\mu A^\nu \nabla_\mu A_\nu$

Must Do Physics for Day 3

1. $F_g = -Gm\psi\hat{R}$ Like charges attract.

2. $+m$ One charge.

3. $\rho = \nabla^2 \phi$ Newton's gravitational field equation.

4. $m \frac{d^2 \vec{R}}{dt^2} = -\frac{GMm}{R^2} \hat{R}$ Newton's law of gravity under classical conditions.

5. $d\tau^2 = (1 - 2\frac{GM}{c^2 R} + 2(\frac{GM}{c^2 R})^2)\, dt^2 - (1 + 2\frac{GM}{c^2 R})\, \frac{dR^2}{c^2}$
   Consistent with the Schwarzschild metric.

6. $F_{\mathrm{EM}} = q\, \vec{E}$ Like charges repel.

7. $\pm\, q$ Two distinct charges.

8. $\rho = \vec{\nabla} \cdot \vec{E} \quad \vec{J} = -\frac{\partial \vec{E}}{c \partial t} + \vec{\nabla} \times \vec{B}$ Maxwell source equations.

9. $0 = \vec{\nabla} \cdot \vec{B} \quad \vec{0} = \frac{\partial \vec{B}}{c \partial t} + \vec{\nabla} \times \vec{E}$ Maxwell homogeneous equations.

10. $F^\mu = q\, \frac{U_\nu}{c}\, (\nabla^\mu A^\nu - \nabla^\nu A^\mu)$ Lorentz force.

11. Unified field emission modes can be quantized.

12. Works with the standard model.

13. Indicates origin of mass.

14. LIGO (gravity wave polarization).Rotation profiles of spiral galaxies.

15. Rotation profiles of spiral galaxies.

16. Big Bang constant velocity distribution.

Field Equation Solutions

1. Newtonian gravitational field equation solution in a vacuum.

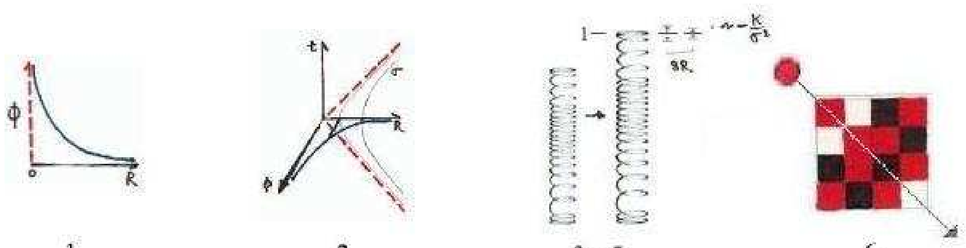
2. 4D Wave equation solution in a vacuum

3. Normalization and perturbations.

4. Normalized perturbation solution to the 4D wave equation.

5. Derivative of the normalized perturbation solution.

6. Only weak gravity.



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Newtonian Gravitational Field Equation Solution in a Vacuum

1. Start with the Newtonian graviational field equations, no source:

   $\nabla^2 \phi = 0$

2. Guess a solution.

   $\phi = (x^2 + y^2 + z^2)^{-\frac{1}{2}} = \frac{1}{R}$

3. Take derivatives:

   $\frac{\partial}{\partial x}(x^2 + y^2 + z^2)^{-\frac{1}{2}} = -x(x^2 + y^2 + z^2)^{-\frac{3}{2}}$

   $\frac{\partial}{\partial y}(x^2 + y^2 + z^2)^{-\frac{1}{2}} = -y(x^2 + y^2 + z^2)^{-\frac{3}{2}}$

   $\frac{\partial}{\partial z}(x^2 + y^2 + z^2)^{-\frac{1}{2}} = -z(x^2 + y^2 + z^2)^{-\frac{3}{2}}$

4. Take second derivatives:

   $\frac{\partial}{\partial x}(-x(x^2 + y^2 + z^2)^{-\frac{3}{2}}) = -(x^2 + y^2 + z^2)^{-\frac{3}{2}} + 3x^2(x^2 + y^2 + z^2)^{-\frac{5}{2}}$

   $\frac{\partial}{\partial y}(-y(x^2 + y^2 + z^2)^{-\frac{3}{2}}) = -(x^2 + y^2 + z^2)^{-\frac{3}{2}} + 3y^2(x^2 + y^2 + z^2)^{-\frac{5}{2}}$

   $\frac{\partial}{\partial z}(-z(x^2 + y^2 + z^2)^{-\frac{3}{2}}) = -(x^2 + y^2 + z^2)^{-\frac{3}{2}} + 3z^2(x^2 + y^2 + z^2)^{-\frac{5}{2}}$

5. Sum:

   $\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0$ QED

- $\vec{\nabla}\frac{1}{R} = -\frac{1}{R^2}\hat{R}$ Practical value: Path to Newton's force law, no source.
- $R = 0$ Practical problem: Point singularity.

4D Wave Equation Solution in a Vacuum

1. Start with 4D wave equation, no source:

   $\Box^2 A^\nu = 0$

2. Guess a solution with similarities to previous:

   $A^\nu = \frac{\sqrt{G}\,h}{c^2}\,((x^2+y^2+z^2-c^2t^2)^{-1}, 0, 0, 0) = \frac{\sqrt{G}\,h}{c^2}\,(\frac{1}{\sigma^2}, \vec{0})$

3. Take derivatives:

   $\frac{\partial}{c\partial t}(x^2+y^2+z^2-c^2t^2)^{-1} = +2c^2\,t\,(x^2+y^2+z^2-c^2t^2)^{-2}$

   $\frac{\partial}{\partial x}(x^2+y^2+z^2-c^2t^2)^{-1} = -2x\,(x^2+y^2+z^2-c^2t^2)^{-2}$

   $\frac{\partial}{\partial y}(x^2+y^2+z^2-c^2t^2)^{-1} = -2y\,(x^2+y^2+z^2-c^2t^2)^{-2}$

   $\frac{\partial}{\partial z}(x^2+y^2+z^2-c^2t^2)^{-1} = -2z\,(x^2+y^2+z^2-c^2t^2)^{-2}$

4. Take second derivatives:

   $\frac{\partial}{c\partial t}(+2c^2\,t\sigma^{-4}) = +2c^2(x^2+y^2+z^2-c^2t^2)^{-2} + 8c^4\,t^2(x^2+y^2+z^2-c^2t^2)^{-3}$

   $\frac{\partial}{\partial x}(-2x\sigma^{-4}) = -2(x^2+y^2+z^2-c^2t^2)^{-2} + 8x^2(x^2+y^2+z^2-c^2t^2)^{-3}$

   $\frac{\partial}{\partial y}(-2y\sigma^{-4}) = -2(x^2+y^2+z^2-c^2t^2)^{-2} + 8y^2(x^2+y^2+z^2-c^2t^2)^{-3}$

   $\frac{\partial}{\partial z}(-2z\sigma^{-4}) = -2(x^2+y^2+z^2-c^2t^2)^{-2} + 8z^2(x^2+y^2+z^2-c^2t^2)^{-3}$

5. Sum:

   $\frac{\partial^2 A_0}{c^2\partial t^2} - \frac{\partial^2 A_0}{\partial x^2} - \frac{\partial^2 A_0}{\partial y^2} - \frac{\partial^2 A_0}{\partial z^2} = 0$ QED

- $x^2+y^2+z^2-c^2t^2 = 0$ Practical value: Singularity is the lightcone.

- $\vec{\nabla}\frac{1}{x^2+y^2+z^2-c^2t^2} \neq f(\frac{1}{R^2})$ Practical problem: Derivative is not an inverse square law.

Normalization and Perturbations

Quantum mechanics cliché: normalize and look at perturbations for a weak field. Gravity is weak.

1. Normalization:

   - U(1)xSU(2)xSU(3) Unitary requirement of the standard model.

- $\frac{A^{\mu}}{|A^{\mu}|} \rightsquigarrow -$ Dimensionless.

2. Perturbations:

   - $A \longrightarrow A' = A + k\delta$ Linear restoration.
   - k Spring constant (small number).
   - δ Variable.

Normalized, Perturbation Solution

1. Start with 4D wave equation solution:

$$A^{\nu} = \frac{\sqrt{G}\,h}{c^2}\,(\frac{1}{x^2+y^2+z^2-c^2t^2}, \vec{0}) = \frac{\sqrt{G}\,h}{c^2}\,(\frac{1}{\sigma^2}, \vec{0})$$

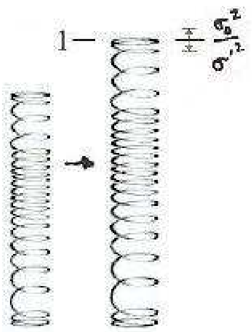
2. Normalize so that the magnitude of A^{μ} is equal to one:

$$A^{\nu} = \frac{A^{\nu}}{|A^{\nu}|} = \frac{c}{\sqrt{G}}\,(\frac{1}{\frac{x^2+y^2+z^2-c^2t^2}{\sigma^2}}, \vec{0}) = (1, \vec{0})$$

3. Perturb $x, y, z,$ and t linearly with a spring constant k:

$$A^{\nu} = \frac{A'^{\nu}}{|A^{\nu}|} = \frac{c}{\sqrt{G}}\,(\frac{1}{(\frac{1}{\sqrt{2}}+\frac{kx}{\sigma^2})^2+(\frac{1}{\sqrt{2}}+\frac{ky}{\sigma^2})^2+(\frac{1}{\sqrt{2}}+\frac{kz}{\sigma^2})^2-(\frac{1}{\sqrt{2}}+\frac{kct}{\sigma^2})^2}, \vec{0})$$

$$= \frac{c}{\sqrt{G}}\,(\sim 1, \vec{0}) = \frac{c}{\sqrt{G}}\,(\frac{\sigma^2}{\sigma'^2}, \vec{0})$$



Derivative of the Normalized, Perturbation Solution

1. Start with the normalized, perturbation solution:

$$A^{\nu} = \frac{A'^{\nu}}{|A^{\nu}|} = \frac{c}{\sqrt{G}}\,(\frac{1}{(\frac{1}{\sqrt{2}}+\frac{kx}{\sigma^2})^2+(\frac{1}{\sqrt{2}}+\frac{ky}{\sigma^2})^2+(\frac{1}{\sqrt{2}}+\frac{kz}{\sigma^2})^2-(\frac{1}{\sqrt{2}}+\frac{kct}{\sigma^2})^2}, \vec{0}) = \frac{c}{\sqrt{G}}\,(\frac{\sigma^2}{\sigma'^2}, \vec{0})$$

2. Expand:

$$A^\nu = \frac{c}{\sqrt{G}} \Big(\frac{1}{(\frac{1}{2} + \frac{k x}{\sqrt{2}\,\sigma^2} + \frac{k^2 x^2}{\sigma^4}) + (\frac{1}{2} + \frac{k y}{\sqrt{2}\,\sigma^2} + \frac{k^2 y^2}{\sigma^4}) + (\frac{1}{2} + \frac{k z}{\sqrt{2}\,\sigma^2} + \frac{k^2 z^2}{\sigma^4}) - (\frac{1}{2} + \frac{k c t}{\sqrt{2}\,\sigma^2} + \frac{k^2 c^2 t^2}{\sigma^4})},$$

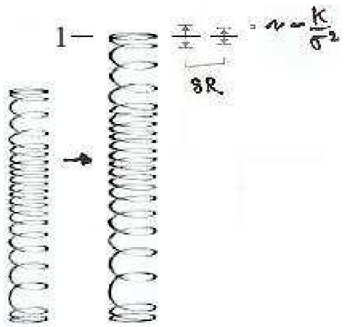
$$\vec{0}\Big) = \frac{c}{\sqrt{G}} \Big(\frac{\sigma_0^2}{\sigma'^2}, \vec{0}\Big)$$

3. Take derivatives:

$$\frac{\partial A^\nu}{c\,\partial t} = \frac{c^2}{\sqrt{G}}\,\frac{\sigma_0^2}{\sigma'^4}\,k + O(k^2) \cong \frac{c^2}{\sqrt{G}}\,\frac{k}{\sigma^2} + O(k^2)$$ where the $\frac{1}{\sqrt{2}}$ is now part of k

$$\frac{\partial A^\nu}{\partial R} = -\frac{c^2}{\sqrt{G}}\,\frac{\sigma_0^2}{\sigma'^4}\,k + O(k^2) \cong -\frac{c^2}{\sqrt{G}}\,\frac{k}{\sigma^2} + O(k^2)$$

- $\frac{1}{\sigma^2}$ — An inverse square distance dependence.
- k — A small number with units of distance.



Only Weak Gravity

A potential that only applies to gravity not EM will have a diagonal field strength tensor.

- The sign of the spring constant k does not effect solving the field equations.
- The sign of the spring constant k does change the derivative of the potential to first order in k.
- Therefore a potential that only has derivatives along the diagonal can be constructed from two potentials that differ by string constants that either constructively interfere to create a non-zero derivative, or destructively interfere to eliminate a derivative.

Diagonal SHO $\quad A^\nu = \frac{c^2}{\sqrt{G}}$

$$\Big(\frac{1}{(\frac{1}{\sqrt{2}} + \frac{k x}{\sigma^2})^2 + (\frac{1}{\sqrt{2}} + \frac{k y}{\sigma^2})^2 + (\frac{1}{\sqrt{2}} + \frac{k z}{\sigma^2})^2 - (\frac{1}{\sqrt{2}} + \frac{k c t}{\sigma^2})^2} + \frac{1}{(\frac{1}{\sqrt{2}} - \frac{k x}{\sigma^2})^2 + (\frac{1}{\sqrt{2}} - \frac{k y}{\sigma^2})^2 + (\frac{1}{\sqrt{2}} - \frac{k z}{\sigma^2})^2 - (\frac{1}{\sqrt{2}} + \frac{k c t}{\sigma^2})^2},$$

$$\frac{1}{(\frac{1}{\sqrt{2}} + \frac{k x}{\sigma^2})^2 + (\frac{1}{\sqrt{2}} + \frac{k y}{\sigma^2})^2 + (\frac{1}{\sqrt{2}} + \frac{k z}{\sigma^2})^2 - (\frac{1}{\sqrt{2}} + \frac{k c t}{\sigma^2})^2} + \frac{1}{(\frac{1}{\sqrt{2}} + \frac{k x}{\sigma^2})^2 + (\frac{1}{\sqrt{2}} - \frac{k y}{\sigma^2})^2 + (\frac{1}{\sqrt{2}} - \frac{k z}{\sigma^2})^2 - (\frac{1}{\sqrt{2}} - \frac{k c t}{\sigma^2})^2},$$

$$\frac{1}{(\frac{1}{\sqrt{2}}+\frac{kx}{\sigma^2})^2+(\frac{1}{\sqrt{2}}+\frac{ky}{\sigma^2})^2+(\frac{1}{\sqrt{2}}+\frac{kz}{\sigma^2})^2-(\frac{1}{\sqrt{2}}+\frac{kct}{\sigma^2})^2}+\frac{1}{(\frac{1}{\sqrt{2}}-\frac{kx}{\sigma^2})^2+(\frac{1}{\sqrt{2}}+\frac{ky}{\sigma^2})^2+(\frac{1}{\sqrt{2}}-\frac{kz}{\sigma^2})^2-(\frac{1}{\sqrt{2}}-\frac{kct}{\sigma^2})^2},$$

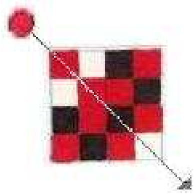
$$\frac{1}{(\frac{1}{\sqrt{2}}+\frac{kx}{\sigma^2})^2+(\frac{1}{\sqrt{2}}+\frac{ky}{\sigma^2})^2+(\frac{1}{\sqrt{2}}+\frac{kz}{\sigma^2})^2-(\frac{1}{\sqrt{2}}+\frac{kct}{\sigma^2})^2}+\frac{1}{(\frac{1}{\sqrt{2}}-\frac{kx}{\sigma^2})^2+(\frac{1}{\sqrt{2}}-\frac{ky}{\sigma^2})^2+(\frac{1}{\sqrt{2}}+\frac{kz}{\sigma^2})^2-(\frac{1}{\sqrt{2}}-\frac{kct}{\sigma^2})^2})$$

Notice the pattern for signs of k.

Take the covariant derivative of this potential, keeping only the terms to first order in the spring constant k.

$$\nabla^\mu A^\nu \cong \frac{c^2\,k}{\sqrt{G}\,\sigma^2}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

This is an Minkowski matrix times $\frac{c^2}{\sqrt{G}}\,\frac{k}{\sigma 2}$, a simple end result that required much work.

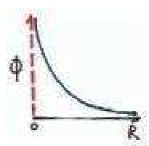


Summary: Weak Field Solution

Math:

$$\nabla^\mu A^\nu \cong \frac{c^2\,k}{\sqrt{G}\,\sigma^2}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Pictures:



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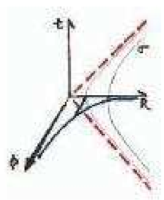

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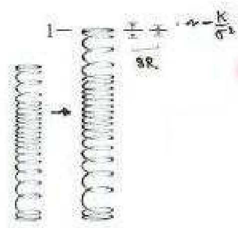

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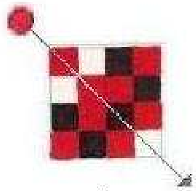

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Stresses, Forces, and Geodesics

1. Stresses:

    a) Stress tensor.

    b) Stress tensor of GEM.

2. Forces:

    a) EM Lorentz force.

    b) EM to gravity analogy.

    c) Gravitational force.

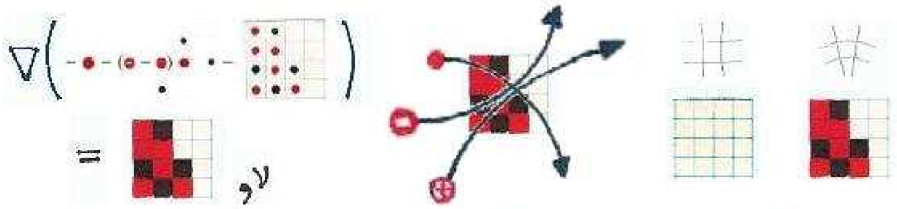
    d) GEM force.

3. Geodesics:

    a) Effect of a geodesic.

    b) Cause of curvature in a geodesic.

    c) Killing's differential equation.



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Stress Tensor

The rank-2 stress tensor is related to a derivative of a Lagrange density.

1. Start with a Lagrange density:

   $\mathfrak{L} = f(A_\sigma, \nabla_\mu A_\sigma)$

2. Take the derivative:

   $\nabla^\nu \mathfrak{L} = \frac{\partial \mathfrak{L}}{\partial A_\sigma} \nabla^\nu A_\sigma + \frac{\partial \mathfrak{L}}{\partial \nabla_\mu A_\sigma} \nabla^\nu \nabla_\mu A_\sigma$

3. Use the Euler-Lagrange equation on the first term, $\frac{\partial \mathfrak{L}}{\partial A_\sigma} = \nabla_\mu (\frac{\partial \mathfrak{L}}{\partial \nabla_\mu A_\sigma})$.

   Change the order of partial derivatives in the second term:

   $\nabla^\nu \mathfrak{L} = \nabla_\mu (\frac{\partial \mathfrak{L}}{\partial \nabla_\mu A_\sigma}) \nabla^\nu A_\sigma + \frac{\partial \mathfrak{L}}{\partial \nabla_\mu A_\sigma} \nabla_\mu \nabla^\nu A_\sigma$

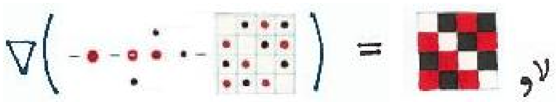
4. Apply the chain rule to condense into one term:

$$\nabla^{\nu}\mathfrak{L} = \nabla_{\mu}\left(\left(\frac{\partial \mathfrak{L}}{\partial \nabla_{\mu} A_{\sigma}}\right)\nabla^{\nu} A_{\sigma}\right)$$

5. Define the rank-2 stress tensor as the stuff inside,

   minus the Lagrange density:

   $$T^{\mu\,\nu} \equiv \left(\frac{\partial \mathfrak{L}}{\partial \nabla_{\mu} A_{\sigma}}\right)\nabla^{\nu} A_{\sigma} - g^{\mu\,\nu}\mathcal{L}$$



Stress Tensor of GEM

1. Start with the stress tensor definition:

   $$T^{\mu\,\nu} \equiv \left(\frac{\partial \mathfrak{L}}{\partial \nabla_{\mu} A_{\sigma}}\right)\nabla^{\nu} A_{\sigma} - g^{\mu\,\nu}\mathcal{L}$$

2. GEM Lagrange density in a vacuum:

   $$\mathfrak{L}_{\text{GEM}} = -\frac{1}{2c^2}\nabla^{\lambda} A^{\sigma}\nabla_{\lambda} A_{\sigma}$$

3. Apply:

   $$T^{\mu\nu} = -\frac{1}{2c^2}\nabla^{\nu} A^{\sigma}\nabla^{\mu} A_{\sigma} + \frac{1}{2c^2} g^{\mu\nu}\nabla^{\lambda} A^{\sigma}\nabla_{\lambda} A_{\sigma}$$

4. Write out the energy density term.

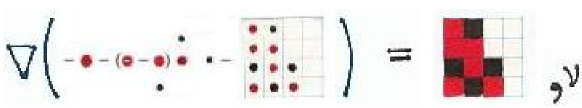
   $$T^{00} = -\frac{1}{c^2}\frac{\partial A^{\sigma}}{\partial t}\frac{\partial A_{\sigma}}{\partial t} + \frac{1}{2c^2} g^{00}\left(\left(\frac{\partial \phi}{\partial t}\right)^2 - (\nabla\phi)^2 - \left(\frac{\partial \vec{A}}{\partial t}\right)^2 + (\nabla\vec{A})^2\right)$$

   $$= -\frac{1}{2c^2}\left(\frac{\partial \phi}{\partial t}\right)^2 + \frac{1}{2c^2}\left(\frac{\partial \vec{A}}{\partial t}\right)^2 - \frac{1}{2}(\nabla\phi)^2 + \frac{1}{2}(\nabla\vec{A})^2$$

   $$= -\frac{1}{2}g_0^2 - \frac{1}{2}eE + \frac{1}{2}B^2 + \frac{1}{2}b^2$$

Notes:

- $T^{00} = E^2 + B^2$ in EM. It is unclear what the difference means.
- $T^{\mu\nu} \longrightarrow F^{\mu}$ There should be a path between the GEM stress tensor and the relativistic force, but I have not figured it out yet.

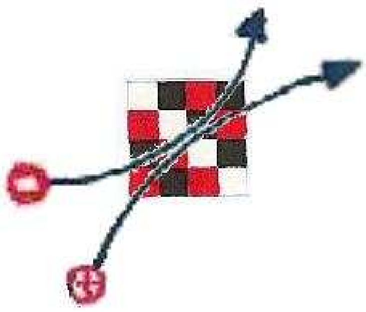


EM Lorentz Force

The Lorentz force is caused by an electric charge moving in an EM field. The effect is to push particles around.

$F^{\mu}_{\rm EM} = q \frac{U_{\nu}}{c} (\nabla^{\mu} A^{\nu} - \nabla^{\nu} A^{\mu}) = \frac{\partial m U^{\mu}}{\partial \tau}$

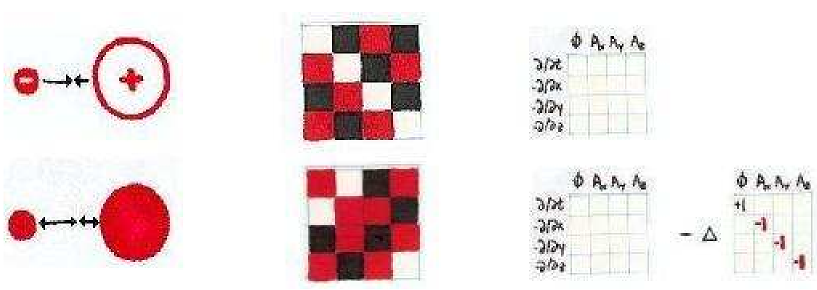
- The cause is electric charge times the velocity contracted with the antisymmetric field strength tensor.
- The effect is to change momentum with respect to the interval τ.
- If the sign of charge is inverted ($q \longrightarrow -q$), $F^{\mu}_{\rm EM}$ flips signs, so there are two distinguishable electric charges.
- Like electrical charges are forced away from each other due to the positive sign of the force.



EM to Gravity Analogy

- $-q \longrightarrow +\sqrt{G}\, m$ Electric charge to mass charge.
- Change field strength tensor's symmetry.

    1. $A - A \longrightarrow A + A$ Anti-symmetric to symmetric tensor.
    2. $, \longrightarrow ;$ Derivatives to contravariant derivatives.



Gravitational Force

The gravitational force is caused by a mass charge moving in a gravitational field. The effect is to push particles around.

$$F_G^{\mu} = -\sqrt{G}\, m\, \frac{U_{\nu}}{c}(\nabla^{\mu} A^{\nu} + \nabla^{\nu} A^{\mu}) = \frac{\partial m U^{\mu}}{\partial \tau}$$

- The cause is mass charge times the velocity contracted with the symmetric field strength tensor.
- The effect is to change momentum with respect to the interval τ.
- If the sign of mass is inverted ($m \longrightarrow -m$), F_G^{μ} is invariant so there is one distinguishable mass charge.
- Mass charges are forced toward each other due to the negative sign of the force.

GEM Force

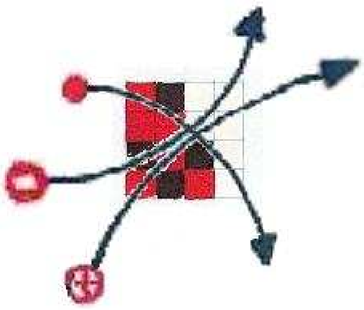
The GEM force is the sum of the gravitational and EM forces.

$$F_{\mathrm{GEM}}^{\mu} = -(\sqrt{G}\, m - q)\, \frac{U_{\nu}}{c} \nabla^{\mu} A^{\nu} - (\sqrt{G}\, m + q)\, \frac{U_{\nu}}{c}\, \nabla^{\nu} A^{\mu} = \frac{\partial m U^{\mu}}{\partial \tau}$$

- $F_{\mathrm{GEM}}^{\mu} = F_G^{\mu}$ if $q = 0$ The GEM force is the gravitational force if the electric charge is zero.
- $F_{\mathrm{GEM}}^{\mu} \longrightarrow F_{\mathrm{EM}}^{\mu}$ as $\frac{\sqrt{G}\, m}{\sqrt{h c}} \longrightarrow 0$

  The GEM force approaches the Lorentz force if the mass charge is small compared to the fundamental electric charge ($n \sqrt{hc}$, where n is an integer for the number of quanta of charges). For one electron:

  $$\sqrt{\frac{6.67x10^{-11}\, \frac{m^3}{\mathrm{kg}\, s^2}}{6.63x10^{-34}\, \frac{\mathrm{kg}\, m^2}{s}\, 3.00x10^{8}\, \frac{m}{s}}}\; 9.11x10^{-31}\, \mathrm{kg} = 1.67x\, 10^{-23}$$



Effect of a Geodesic

A geodesic is the path of zero external force. Investigate the change in momentum (or effect) term of F^{μ}_{GEM}.

1. Start with the change in momentum set equal to zero. Apply the chain rule to expand:

   $$0=\frac{\partial m U^{\mu}}{\partial \tau}=m\,\frac{\partial U^{\mu}}{\partial \tau}+U^{\mu}\,\frac{\partial m}{\partial \tau}$$

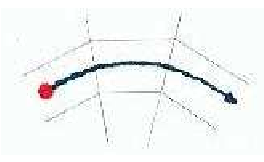
2. Assume $\frac{\partial m}{\partial \tau}=0$. Use the chain rule to expand $\frac{\partial U^{\mu}}{\partial \tau}$:

   $$0=m\,\frac{\partial U^{\mu}}{\partial \tau}=m\,\frac{\partial U^{\mu}}{\partial x^{\nu}}\,\frac{\partial x^{\nu}}{\partial \tau}=m\,\nabla_{\nu}U^{\mu}U^{\nu}$$

3. Apply the definition of a contravariant derivative of a contravariant vector, normal derivative + change in the metric, $\nabla_{\nu}U^{\mu}=\partial_{\nu}U^{\mu}+\Gamma_{\varpi\nu}{}^{\mu}\,U^{\varpi}$:

   $$0=m\partial_{\nu}U^{\mu}U^{\nu}+m\Gamma_{\varpi\nu}{}^{\mu}U^{\omega}U^{\nu}=m\,\frac{\partial^2 x^{\nu}}{\partial \tau^2}+m\,\Gamma_{\varpi\nu}{}^{\mu}\,U^{\omega}U^{\nu}$$

If any acceleration is seen without a force ($m\,\frac{\partial^2 x^{\nu}}{\partial \tau^2}\neq 0, F^{\mu}_{\text{GEM}}=0$), then the effect is entirely due to the curvature of spacetime ($m\,\Gamma_{\varpi\nu}{}^{\mu}\,U^{\omega}U^{\nu}\neq 0$).



Cause of Curvature

Every effect must have a cause. Explore the change in potential (or cause) term.

1. Start with force set equal to zero:

   $$0=-\,(\sqrt{G}\,m-q)\,\frac{U_{\nu}}{c}\,\nabla^{\mu}A^{\nu}-(\sqrt{G}\,m+q)\,\frac{U_{\nu}}{c}\,\nabla^{\nu}A^{\mu}$$

2. Apply the definition of a covariant derivative of a contravariant vector, normal derivative - change in the metric, $\nabla^\mu A^\nu = \partial^\mu A^\nu - \Gamma_\varpi{}^{\mu\nu} A^\varpi$:

$$0 = -(\sqrt{G}\,m - q)\,\frac{U_\nu}{c}\,\partial^\mu A^\nu - (\sqrt{G}\,m + q)\,\frac{U_\nu}{c}\,\partial^\nu A^\mu$$

$$+\sqrt{G}\,m\,\frac{U_\nu}{c}\,\Gamma_\varpi{}^{\mu\nu} A^\varpi - q\,\frac{U_\nu}{c}\,\Gamma_\varpi{}^{\mu\nu} A^\varpi$$

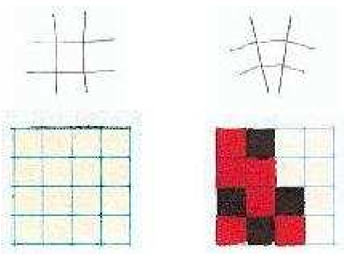
$$+\sqrt{G}\,m\,\frac{U_\nu}{c}\,\Gamma_\varpi{}^{\nu\mu} A^\varpi + q\,\frac{U_\nu}{c}\,\Gamma_\varpi{}^{\nu\mu} A^\varpi$$

$$= -(\sqrt{G}\,m - q)\,\frac{U_\nu}{c}\,\partial^\mu A^\nu - (\sqrt{G}\,m + q)\,\frac{U_\nu}{c}\,\partial^\nu A^\mu + 2\sqrt{G}\,m\,\frac{U_\nu}{c}\,\Gamma_\varpi{}^{\mu\nu} A^\varpi$$

Curvature is coupled directly to mass, not to q.

Curvature of spacetime without a force ($2\sqrt{G}\,m\,\frac{U_\nu}{c}\,\Gamma_\varpi{}^{\mu\nu} A^\varpi \neq 0, F^\mu_{\mathrm{GEM}} = 0$) is caused by change in the potential which are coupled to both the mass charge and electric charge.

General relativity provides a way to calculate curvature by comparing two nearby geodesics using a tidal effect. Because general relativity lacks a means within the geodesic to calculate the cause of curvature, general relativity is incomplete.

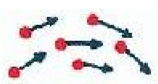


Killing's Differential Equation

If $F^\mu_{\mathrm{GEM}} = 0$, then $\alpha \nabla^\mu A^\nu + \beta \nabla^\nu A^\mu = 0$. This is a generalization of Killing's differential equation where $\alpha = \beta = 1$. The solutions are known as Killing vector fields.

There are two conserved quantities:

- Energy
- Angular momentum

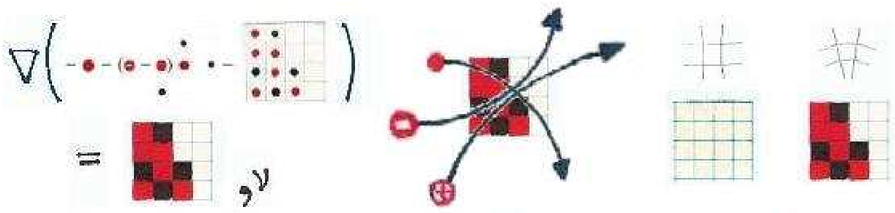


Summary: Stresses, Forces, and Geodesics

Math:

$$F_{\mathrm{GEM}}^{\mu} = -\,(\sqrt{G}\,m - q)\,\frac{U_\nu}{c}\,\nabla^\mu A^\nu - (\sqrt{G}\,m + q)\,\frac{U_\nu}{c}\,\nabla^\mu A^\nu = \frac{\partial\, m\, U^\mu}{\partial \tau}$$

Pictures:



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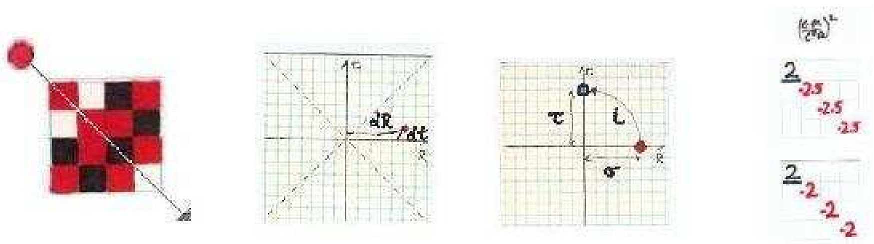
Relativistic Gravitational Force

1. Weak field approximation.
2. Exact solution.
3. Exact solution applied.
4. Schwarzschild metric.
5. Schwarzschild versus GEM metric.



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Weak Field Approximation

1. Start from the gravitational force law:

   $$F_G^{\mu} = -\,\sqrt{G}\,m\,\frac{U_\nu}{c}(\nabla^\mu A^\nu + \nabla^\nu A^\mu) = \frac{\partial\, m\, U^\mu}{\partial \tau}$$

2. Recall weak gravitational field strength tensor which assumes the field is electrically neutral and weak:

   $$\nabla^\mu A^\nu \cong \frac{c^2\, k}{\sqrt{G}\,\sigma^2}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

3. Check units of $\nabla^\mu A^\nu$ to the derivative of the normalized potential:

$$\sqrt{G}\, m \nabla^\mu A^\nu \rightsquigarrow \frac{\sqrt{L^3}}{t\sqrt{m}}\, m\, \frac{\sqrt{m}}{t\sqrt{L}} = \frac{mL}{t^2}$$

$$cm\, \frac{\partial \frac{A'^\mu}{|A^\mu|}}{\partial t} \rightsquigarrow \frac{L}{t}\, m\, \frac{1}{t} = \frac{mL}{t^2}$$

4. Substitute the normalized potential derivative into the force law, noting the units and the sign flip on the contravariant derivative. Expand the velocities, $U_\nu \rightarrow (U_0, -\vec{U})$ and $U^\mu \rightarrow (U_0, \vec{U})$:

$$F_G^\mu = -mc^2 \left(\frac{U_0}{c}, -\frac{\vec{U}}{c}\right) \begin{pmatrix} \frac{k}{\sigma^2} & 0 \\ 0 & \frac{k}{\sigma^2} \end{pmatrix} = \left(\frac{\partial m U_0}{\partial \tau}, \frac{\partial m \vec{U}}{\partial \tau}\right)$$

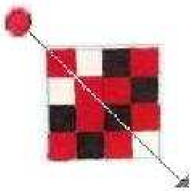
5. Contract the rank-1 velocity tensor with the rank-2 derivative of the potential:

$$F_G^\mu = m\left(-\frac{ck}{\sigma^2} U_0, \frac{ck}{\sigma^2} \vec{U}\right) = \left(\frac{\partial m U_0}{\partial \tau}, \frac{\partial m \vec{U}}{\partial \tau}\right)$$

6. Substitute $c^2\tau^2$ for $-\sigma^2$:

$$F_G^\mu = m\left(\frac{k}{\tau^2} \frac{U_0}{c}, -\frac{k}{\tau^2} \frac{\vec{U}}{c}\right) = \left(\frac{\partial m U_0}{\partial \tau}, \frac{\partial m \vec{U}}{\partial \tau}\right)$$

Warning: The relationship between σ^2 and τ^2 is simple. What gets tricky is the relationship between σ and τ, because there the signs are "free" ($\pm i\sigma \rightarrow \pm c\tau$).



Exact Solution

The gravitational force for the weak field is a first order differential equation that can be solved exactly.

1. Start from the gravitational force for a weak field:

$$F_G^\mu = m\left(\frac{k}{\tau^2} \frac{U_0}{c}, -\frac{k}{\tau^2} \frac{\vec{U}}{c}\right) = \left(\frac{\partial m U_0}{\partial \tau}, \frac{\partial m \vec{U}}{\partial \tau}\right)$$

2. Apply the chain rule to the cause terms. Assume $U_0 \frac{\partial m}{\partial \tau} = \vec{U} \frac{\partial m}{\partial \tau} = 0$.

   Collect terms on one side:

$$\left(m \frac{\partial U_0}{\partial \tau} - m\frac{k}{\tau^2} \frac{U_0}{c}, m \frac{\partial \vec{U}}{\partial \tau} + m \frac{k}{\tau^2} \frac{\vec{U}}{c}\right) = 0$$

3. Assume the equivalence principle. Drop m:

   $$(\frac{\partial U_0}{\partial \tau} - \frac{k}{\tau^2}\frac{U_0}{c}, \frac{\partial \vec{U}}{\partial \tau} + \frac{k}{\tau^2}\frac{\vec{U}}{c}) = 0$$

4. Solve for velocity:

   $$(U_0, \vec{U}) = (c_0\, e^{-\frac{k}{c\tau}}, \vec{C}_{1-3}\, e^{+\frac{k}{c\tau}})$$

5. Contract the velocity solution:

   $$U^\mu U_\mu = c_0^2\, e^{-2\frac{k}{c\tau}} - \vec{C}_{1-3}\, e^{+2\frac{k}{c\tau}}$$

6. For flat spacetime ($k \to 0, \text{or}\; \tau \to \infty$), there are four constraints on the contracted velocity solution:

   $$U^\mu U_\mu = (c\frac{\partial t}{\partial \tau}, \frac{\partial \vec{R}}{\partial \tau})(c\frac{\partial t}{\partial \tau}, -\frac{\partial \vec{R}}{\partial \tau}) = \frac{c^2(\partial t)^2 - (\partial R)^2}{(\partial t)^2 - (\frac{\partial R}{c})^2} = c^2$$

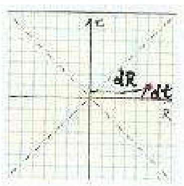
   True if and only if: $c_0^2 = c\frac{\partial t}{\partial \tau} = U_{0\,\text{flat}}, \quad \vec{C}_{1-3} = \frac{\partial \vec{R}}{\partial \tau} = \vec{U}_{\text{flat}}$

7. Substitute $c\frac{\partial t}{\partial \tau}$ for c_0^2, $\frac{\partial \vec{R}}{\partial \tau}$ for $\vec{C}_{1-3}$ into the contracted velocity solution. Multiply through by $(\frac{\partial \tau}{c})^2$:

   $$(\partial \tau)^2 = e^{-2\frac{k}{c\tau}}(\partial t)^2 - e^{+2\frac{k}{c\tau}}(\frac{\partial \vec{R}}{c})^2$$

This is a unique algebraic road to a metric equations. The logic will have to be looked at by mathematicians.

- $k = 0$, or $\tau \to \infty$ Flat spacetime.
- $e^{-2\frac{k}{c\tau}} \neq 1$ Curved spacetime.



Exact Solution Applied

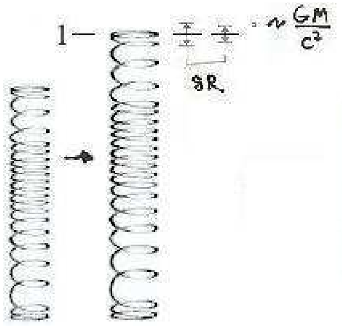
Apply to a weak, spherically symmetric, gravitational system.

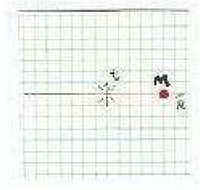
- $k = \frac{GM}{c^2} \rightsquigarrow \frac{L^3}{mt^2} m \frac{t^2}{L^2} = L$ Gravitational source spring constant.
- $\sigma^2 = R^2 - (ct)^2 = R'^2$ Static field approximated by R'.

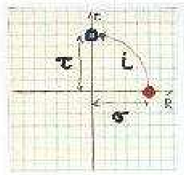
- $|\sigma| = |c\tau| = R$ σ and $c\tau$ have the same magnitude.
- $(+i\sigma)^2 = (+c\tau)^2$ To make a real metric, choose σ to be imaginary.

Plug into the exact solution:

$$(\partial\tau)^2 = e^{-2\frac{GM}{c^2 R}}(\partial t)^2 - e^{+2\frac{GM}{c^2 R}}(\frac{\partial\vec{R}}{c})^2$$







Schwarzschild Metric

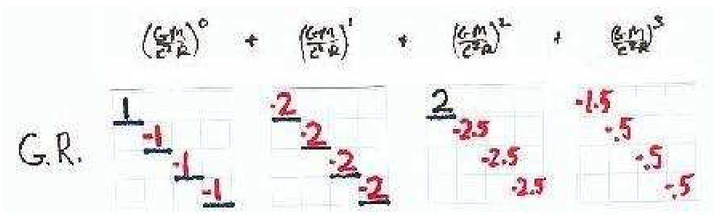
The Schwarzschild metric is a solution of general relativity for a neutral, non-rotating, spherically symmetric source mass (derivation not shown). Write out the Taylor series expansion of the Schwarzschild metric in isotropic coordinates to third order in $\frac{GM}{c^2 R}$.

Schwarzschild metric:

$$(\partial\tau)^2 = (\underline{1 - 2\frac{GM}{c^2 R} + 2(\frac{GM}{c^2 R})^2} - \frac{3}{2}(\frac{GM}{c^2 R})^3)(\partial t)^2 - (\underline{1 - 2\frac{GM}{c^2 R}} + \frac{3}{2}(\frac{GM}{c^2 R})^2 + \frac{1}{2}(\frac{GM}{c^2 R})^3)(\frac{\partial\vec{R}}{c})^2$$

The five underlined terms have been confirmed experimentally. Tests include:

- Light bending around the Sun.
- Perihelion shift of Mercury.
- Time delay in radar reflections off of planets.



Compare Metrics: Schwarzschild to GEM

Write out the Taylor series expansion of the Schwarzschild and GEM metrics in isotropic coordinates to third order in $\frac{GM}{c^2R}$.

1. Schwarzschild metric:

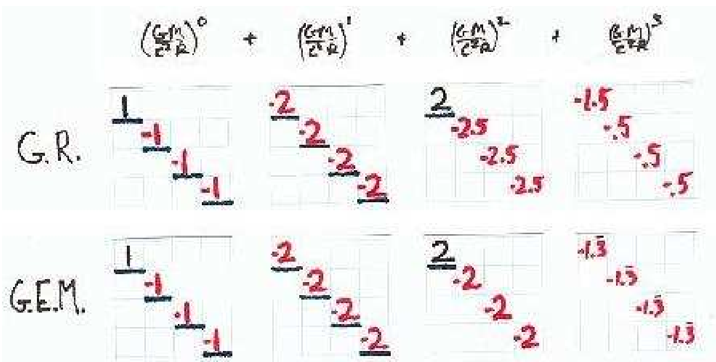
$$(\partial\tau)^2 = (\underline{1 - 2\frac{GM}{c^2R} + 2(\frac{GM}{c^2R})^2} - \frac{3}{2}(\frac{GM}{c^2R})^3)(\partial t)^2 - (\underline{1 - 2\frac{GM}{c^2R}} + \frac{3}{2}(\frac{GM}{c^2R})^2 + \frac{1}{2}(\frac{GM}{c^2R})^3)(\partial\vec{R})^2$$

2. GEM metric:

$$(\partial\tau)^2 = (\underline{1 - 2\frac{GM}{c^2R} + 2(\frac{GM}{c^2R})^2} - \frac{4}{3}(\frac{GM}{c^2R})^3)(\partial t)^2 - (\underline{1 - 2\frac{GM}{c^2R}} + 2(\frac{GM}{c^2R})^2 + \frac{4}{3}(\frac{GM}{c^2R})^3)(\frac{\partial\vec{R}}{c})^2$$

Compare the two metrics:

- Identical for tested terms of Taylor series expansion.
- Different for higher order terms, so can be tested (not easy).
- GEM is more symmetric.



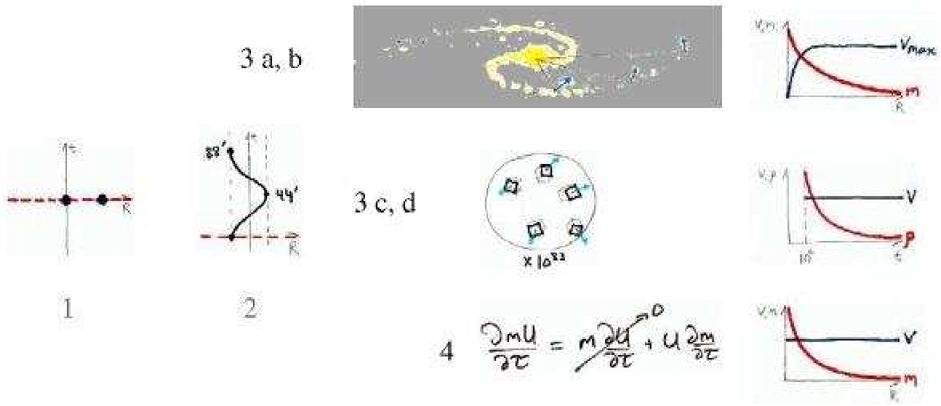
Classical Gravitational Force

1. Breaking spacetime symmetry.
2. Newton's gravitational law derivation.
3. Need for new classical solutions:
   a) Problem statement for rotation profiles of spiral galaxies.
   b) Solution requirements for rotation profiles.

c) Problem statement for the big bang.

d) Solution requirements for the big bang.

4. Constant velocity solutions.

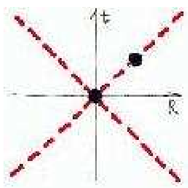


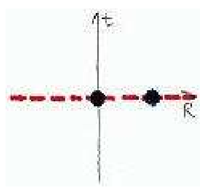
Breaking Spacetime Symmetry

Spacetime symmetry must be broken to go from the relativistic weak gravitational force to a classical force for both cause and effect.

Contrast the relativistic geometry of Minkowski spacetime with the geometry of Newtonian absolute space and time.

Minkowski Spacetime	Geometry	Newtonian Space and Time
True, Elegant	Utility	Accurate, Practical
$(\partial\tau)^2 = (dt)^2 - \frac{(dR)^2}{c^2}$	Interval	$\text{distance}^2 = dR^2 \neq f(t)$
$c = 1$	Speed of Light	$c = \infty$
$(U_0, \vec{U}) = (c\,\frac{\partial t}{\partial\tau}, \frac{\partial\vec{R}}{\partial\tau})$	Velocity	$(\mathbb{U}_0, \vec{\mathbb{U}}) \equiv (\frac{\partial t}{\partial\|R\|}, c\frac{\partial\vec{R}}{\partial\|R\|}) = (0, c\,\hat{R})$
$(\frac{\partial U_0}{\partial\tau}, \frac{\partial\vec{U}}{\partial\tau}) = (c\,\frac{\partial^2 t}{\partial\tau^2}, \frac{\partial^2\vec{R}}{\partial\tau^2})$	Acceleration	$(\frac{\partial\mathbb{U}_0}{\partial\tau}, \frac{\partial\vec{\mathbb{U}}}{\partial\tau}) = (0, c^2\,\frac{\partial^2\vec{R}}{\partial\|R\|^2})$





Newton's Gravitational Law Derivation

1. Start from the relativistic gravitational force for a weak field:

   $F_G^\mu = m\,(\frac{k}{\tau^2}\frac{U_0}{c}, -\frac{k}{\tau^2}\frac{\vec{U}}{c}) = (\frac{\partial\, m U_0}{\partial \tau}, \frac{\partial\, m \vec{U}}{\partial \tau})$

2. Apply the chain rule to the cause terms.

   Assume $U_0 \frac{\partial m}{\partial \tau} = \vec{U}\,\frac{\partial m}{\partial \tau} = 0$:

   $F_G^\mu = m\,(\frac{k}{\tau^2}\frac{U_0}{c}, -\frac{k}{\tau^2}\frac{\vec{U}}{c}) = (m\,\frac{\partial U_0}{\partial \tau}, m\,\frac{\partial \vec{U}}{\partial \tau})$

3. Break spacetime symmetry:

   - $(U_0, \vec{U}) \longrightarrow (\mathbb{U}_0, \vec{\mathbb{U}}) = (0, c\,\hat{R})$
   - $(\frac{\partial U_0}{\partial \tau}, \frac{\partial \vec{U}}{\partial \tau}) \longrightarrow (0, c^2\,\frac{\partial^2 \vec{R}}{\partial |R|^2})$

   $F_G^\mu = m\,(0, -\frac{k}{\tau^2}\,\hat{R}) = (0, m c^2\,\frac{\partial^2 \vec{R}}{\partial |R|^2})$

4. Assume the gravitational spring constant $(k = \frac{GM}{c^2})$:

   $F_G^\mu = (0, -\frac{GMm}{c^2 \tau^2}\,\hat{R}) = (0, m c^2\,\frac{\partial^2 \vec{R}}{\partial |R|^2})$

5. Substitute: σ^2 for $-c^2\,\tau^2$ in the cause term.

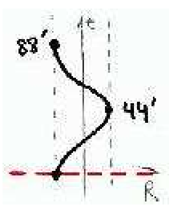
   Substitute: $-c^2\,(\frac{\partial}{\partial \tau})^2$ for $(\frac{\partial}{\partial |R|})^2 = (\frac{\partial}{\partial \sigma})^2$ in the effect term.

   $F_G^\mu = (0, \frac{GMm}{\sigma^2}\,\hat{R}) = (0, -m\,\frac{\partial^2 \vec{R}}{\partial \tau^2})$

6. Assume the static field approximation: $\sigma^2 = R^2 - t^2 \cong R'^2$.

   Assume the low speed approximation: $\frac{\partial^2}{\partial \tau^2} \cong \frac{\partial^2}{\partial t^2}$:

   $F_G^\mu = (0, \frac{GMm}{R^2}\,\hat{R}) = (0, -m\,\frac{\partial^2 \vec{R}}{\partial t^2})$ QED



Problem Statement for the Rotation Profile of Galaxies

The momentum of stars in thin spiral galaxies has two problems:

- The flat velocity profile problem.

  After attaining a maximal speed consistent with Newton's law of gravity near the core, the velocity profile stays flat with increasing distance. Newton's law predicts a "Keplerian" decline for the velocity of the outer stars.

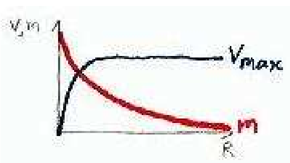
- The stability problem.

  Thin spiral galaxies are mathematically unstable to small disturbances along the axis which should lead to collapse.

Solution Requirements for Rotation Profiles

Requirements for a solution:

1. Stable mathematically to axial perturbations.
2. Same velocity for all outer stars.
3. Describes the change in mass distribution in spacetime, which falls off exponentially with distance ($2 \times 3 = \Delta$momentum).
4. Fits every observational constraint.



Problem Statement for the Big Bang

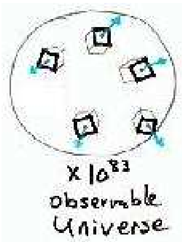
Big bang cosmology has two big problems:

- The horizon problem.

All $\sim 10^{83}$ separate, independent spacetime volumes of the early Universe must travel at the same velocity to create the uniform black body radiation spectrum seen in the cosmic background radiation.

- The flatness problem.

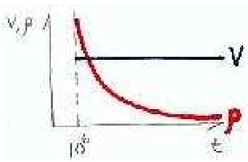
  The initial conditions must be tuned to one part in $\sim 10^{55}$ so the mathematically unstable solution lasts 10^{10} years.



Solution Requirements for the Big Bang

Requirements for a solution:

1. Stable mathematically for initial conditions.
2. Same velocity for all independent regions of spacetime.
3. Describes the change in mass distribution in spacetime, from high density early to lower later ($2 \times 3 = \Delta$momentum).
4. Fits every observational constraint.



Disclosure: I do not know the actual shape of mass density decrease.

Stable Constant Velocity Solutions

1. Start from the gravitational force for a weak field:

   $F_G^{\mu} = m\left(\frac{k}{\tau^2}\frac{U_0}{c}, -\frac{k}{\tau^2}\frac{\vec{U}}{c}\right) = \left(\frac{\partial m U_0}{\partial \tau}, \frac{\partial m \vec{U}}{\partial \tau}\right)$

2. Apply the chain rule to the cause terms.

Assume $m\frac{\partial U_0}{\partial \tau} = m\frac{\partial \vec{U}}{\partial \tau} = 0$ (meaning assume velocity is constant):

$F_G^{\mu} = m(\frac{k}{\tau^2}\frac{U_0}{c}, -\frac{k}{\tau^2}\frac{\vec{U}}{c}) = (U_0\frac{\partial m}{\partial \tau}, \vec{U}\frac{\partial m}{\partial \tau})$

3. Break spacetime symmetry: $(U_0, \vec{U}) \longrightarrow (\mathbb{U}_0, \vec{\mathbb{U}}) = (0, c\hat{R})$.

   $F_G^{\mu} = m(0, -\frac{k}{\tau^2}\hat{R}) = (0, \frac{\partial m}{\partial \tau}c\hat{R})$

4. Assume the gravitational spring constant $(k = \frac{GM}{c^2})$:

   $F_G^{\mu} = (0, -\frac{GMm}{c^2\tau^2}\hat{R}) = (0, \frac{\partial m}{\partial \tau}c\hat{R})$

5. Collect terms on one side:

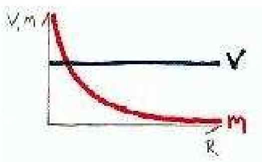
   $(c\frac{\partial m}{\partial \tau} + \frac{GMm}{c^2\tau^2})(0, \hat{R}) = 0$

6. Solve for m:

   $m = m_o e^{\frac{GM}{c^3\tau}}$

7. Substitute: R for $c\tau$ which depends on exactly the same assumptions used in the metric derivation (static field, $|\sigma| = |\tau|$=R, and sigma is imaginary):

   $m = m_0 e^{\frac{GM}{c^2 R}}$



Must Do Physics Done for Day 3

1. $F_g = -Gm\psi\hat{R}$ Like charges attract.
2. $+m$ One charge.
3. $\rho = \nabla^2\phi$ Newton's gravitational field equation.
4. $m\frac{d^2\vec{R}}{dt^2} = -\frac{GMm}{R^2}\hat{R}$ Newton's law of gravity under classical conditions.
5. $d\tau^2 = (1 - 2\frac{GM}{c^2R} + 2(\frac{GM}{c^2R})^2)dt^2 - (1 + 2\frac{GM}{c^2R})\frac{dR^2}{c^2}$ Consistent with the Schwarzschild metric.
6. $F_{\mathrm{EM}} = q\vec{E}$ Like charges repel.

7. $\pm q$ — Two distinct charges.

8. $\rho = \vec{\nabla} \cdot \vec{E} \quad \vec{J} = -\frac{\partial \vec{E}}{c\,\partial t} + \vec{\nabla} \times \vec{B}$ — Maxwell source equations.

9. $0 = \vec{\nabla} \cdot \vec{B} \quad \vec{0} = \frac{\partial \vec{B}}{c \partial t} + \vec{\nabla} \times \vec{E}$ — Maxwell homogeneous equations.

10. $F^{\mu} = q\,\frac{U_{\nu}}{c}(\nabla^{\mu} A^{\nu} - \nabla^{\nu} A^{\mu})$ — Lorentz force.

11. Unified field emission modes can be quantized.

12. Works with the standard model.

13. Indicates origin of mass.

14. LIGO (gravity wave polarization).

15. Rotation profiles of spiral galaxies.

16. Big Bang constant velocity distribution.

Caveats:

5. Check metric derivation. Proposal can be confirm/rejected by experiment.

15. Actual, detailed calculations must be compared with data.

16. See 15.

Summary: Day 3

Math:

1. 4D wave equation solution in a vacuum:

   $A^{\nu} = ((x^2 + y^2 + z^2 - c^2 t^2)^{-1}, 0, 0, 0) = (\frac{1}{\sigma^2}, \vec{0})$

2. Normalized, perturbation weak gravitational field strength tensor:

   $$\nabla^{\mu} A^{\nu} \cong \frac{c^2}{\sqrt{G}}\,\frac{k}{\sigma 2}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

3. Stress tensor of GEM:

   $T^{00} = -\frac{1}{2}\,g_0^2 - \frac{1}{2}\,e E + \frac{1}{2}\,B^2 + \frac{1}{2}\,b^2$

4. GEM force:

   $$F^{\mu}_{\text{GEM}} = -\,(\sqrt{G}\,m - q)\,\frac{U_\nu}{c}\nabla^{\mu}A^{\nu} - (\sqrt{G}\,m + q)\,\frac{U_\nu}{c}\,\nabla^{\nu}A^{\mu} = \frac{\partial m U^{\mu}}{\partial\tau}$$

5. Cause of curvature:

   $$0 = -\,(\sqrt{G}\,m - q)\,\frac{U_\nu}{c}\,\partial^{\mu}A^{\nu} - (\sqrt{G}\,m + q)\,\frac{U_\nu}{c}\,\partial^{\nu}A^{\mu} + 2\,\sqrt{G}\,m\,\frac{U_\nu}{c}\,\Gamma_{\varpi}{}^{\mu\nu}\,A^{\varpi}$$

6. Relativistic gravitational force for a weak field:

   $$F^{\mu}_{G} = m\Big(\frac{k}{\tau^2}\,\frac{U_0}{c}, -\,\frac{k}{\tau^2}\,\frac{\vec{U}}{c}\Big) = \Big(\frac{\partial m U_0}{\partial\tau}, \frac{\partial m \vec{U}}{\partial\tau}\Big)$$

7. GEM metric:

   $$(\partial\tau)^2 = e^{-2\,\frac{GM}{c^2\,R}}\,(\partial\,t)^2 - e^{+2\,\frac{GM}{c^2\,R}}\,\Big(\frac{\partial\vec{R}}{c}\Big)^2$$

8. GEM metric Taylor series expansion:

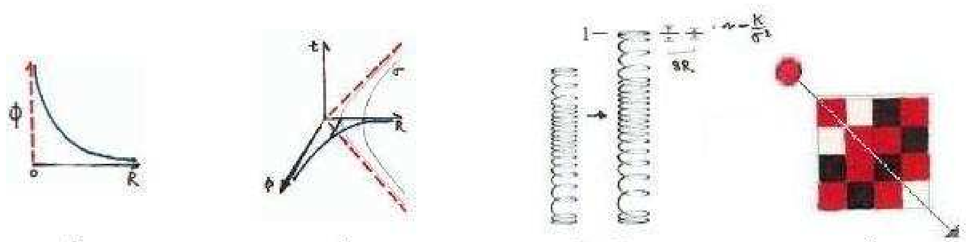
   $$(\partial\tau)^2 = (\underline{1 - 2\,\frac{GM}{c^2\,R} + 2\,(\frac{GM}{c^2\,R})^2} - \frac{4}{3}\,(\frac{GM}{c^2\,R})^3)(\partial\,t)^2$$
   $$-\,(\underline{1 - 2\,\frac{GM}{c^2\,R}} + 2(\frac{GM}{c^2\,R})^2 + \frac{4}{3}(\frac{GM}{c^2\,R})^3)(\frac{\partial\vec{R}}{c})^2$$

9. Stable constant velocity solution:

   $$m = m_0\,e^{\frac{GM}{c^2\,R}}$$

Pictures:

1. Field equation solutions:

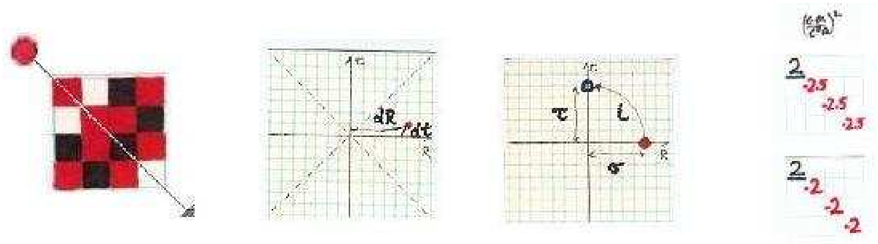


1 2 3 - 5 6

2. Stresses, forces, and geodesics:

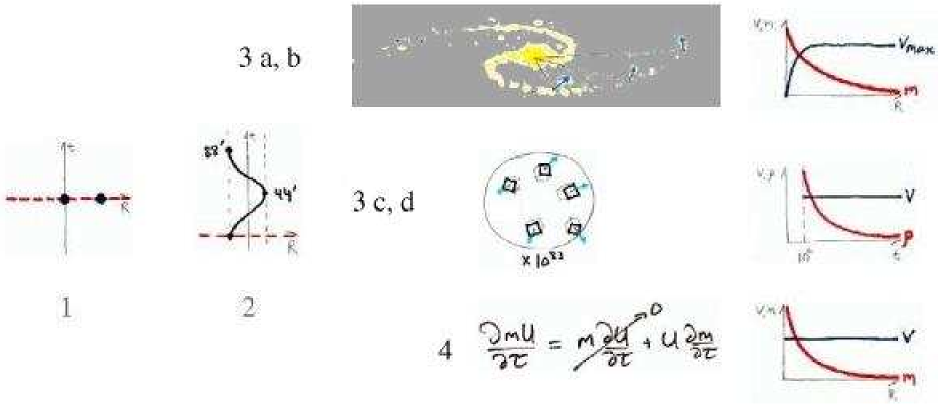
1 2 3

3. Relativistic gravitational force:



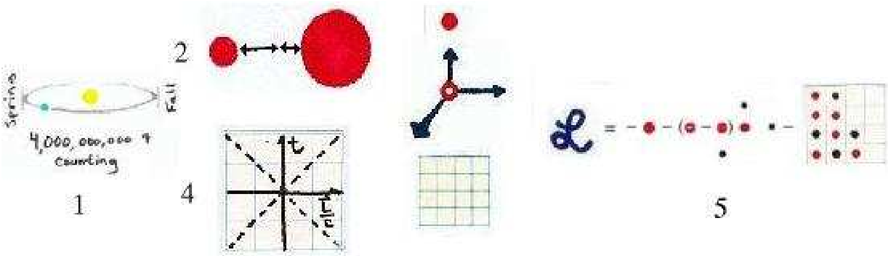
1 2 3 4, 5

4. Classical gravitational force:



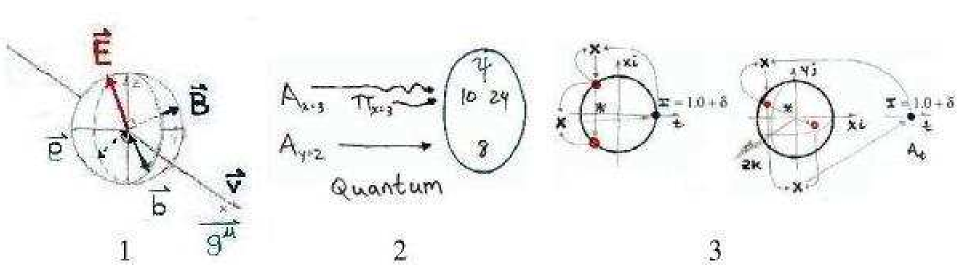
1 2

5. Day 1:



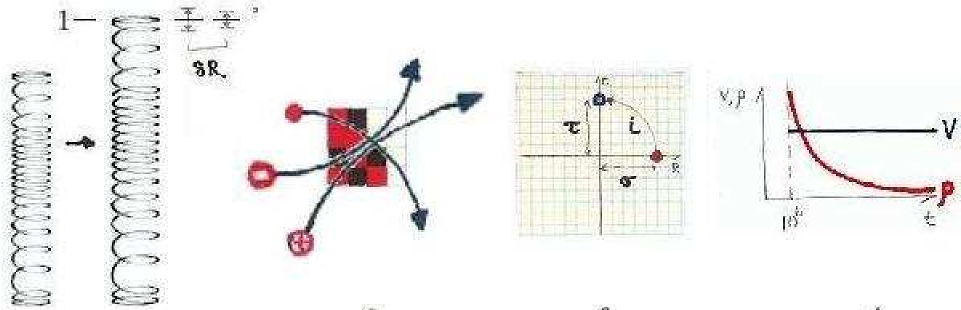
1 3 5

6. Day2:



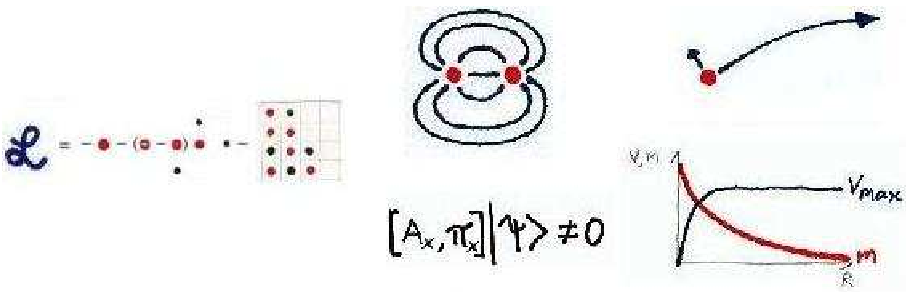
1 2 3

7. Day 3:



1 2 3 4

8. The three lectures:



Day 1 Day 2 Day 3